IX. Games

The Prisoner's Dilemma

		Bob	
		Confess	Not Confess
Ann	Confess	(-10,-10)	(0,-20)
	Not Confess	(-20,0)	(-1,-1)

The first entry in the parentheses is the payoff to the *row player*, Ann, while the second entry is the payoff to the *column player*, Bob.

For example, if Ann confesses and Bob doesn't, Bob loses 20 and Ann gets 0 (goes free), or (0,-20).

Politics

		Bob	
		Lie	Tell Truth
Ann	Believe	(-1,2)	(1,1)
	Disbelieve	(0,0)	(-1,-1)

Nash Equilibrium:

Each player chooses an action that maximizes their (expected) payoff given the action chosen by the other players.

The Battle of the Sexes

The phones are out, so Bob and Ann can't discuss where they should go today, and must independently decide.

		Bob	
		Car Race	Beach
Ann	Car Race	(2,3)	(0,0)
	Beach	(1,1)	(3,2)

Meeting in the Hall

		Bob	
		Left	Right
Ann	Left	(1,1)	(0,0)
	Right	(0,0)	(1,1)

Matching Pennies

Ann wins if there is a match.

		Bob	
		Heads	Tails
Ann	Heads	(1,-1)	(-1,1)
	Tails	(-1,1)	(1,-1)

Chicken

		Bob	
		Swerve	Don't Swerve
Ann	Swerve	(5,5)	(3,10)
	Don't Swerve	(10,3)	(0,0)

Avoiding Bob

Ann wants to avoid Bob; Bob wants to go to the same party as Ann.

		Bob	
		Party 1	Party 2
Ann	Party 1	(-5,5)	(10,0)
	Party 2	(0,-5)	(-10,10)

Bob goes to party 1 with probability p.

If Ann goes to party 1 she gets
$-5p + 10(1-p) = 10 - 15p$

If Ann goes to party 2 she gets
$0p + -10(1-p) = -10 + 10p$

$10 - 15p = -10 + 10p$, or $p = 4/5$.

Ann goes to party 1 with probability q.

If Bob goes to Party 1, he gets
$5q + -5(1-q) = -5 + 10q$

If Bob goes to Party 2, he gets
$0q + 10(1-q) = 10 - 10q$

$-5 + 10q = 10 - 10q$
or, $q = ¼$

Mudslinging

		Republican	
		Clean	Throw Mud
Democrat	Clean	(3,1)	(1,3)
	Throw Mud	(2,1)	(2,0)

Republican Democrat is clean with prob p
If clean, R gets 1; if throws mud, gets
$3p + 0(1-p) = 3p$.
Thus $1 = 3p$ or $p = 1/3$.

Democrat Republican clean with prob q
If clean, D gets $3q+1(1-q)$.
If dirty, D gets 2. Thus
$2 = 3q + (1-q)$, or $q = ½$

Dell Plant Location Game

			Nashville	
		q		1-*q*
		No Concession		Concession
p	No Concession	(3,1)		(1,2)
Austin				
1-*p*	Concession	(2,1)		(2,0)

Nashville: Austin makes no concession with prob *p*
If Nashville doesn't, gets 1;
if Nashville does, gets $2p + 0(1-p) = 2p$.
Thus $1 = 2p$ or $p = ½$

Austin: Nashville makes no concessions with prob *q*
If Austin doesn't, gets $3q+1(1-q)$.
If Austin does, gets 2.
Thus $2 = 3q + (1-q)$, or $q = ½$

Cleaning the Apartment

		Roommate	
		Clean	Don't
	Clean	(10,10)	(0,15)
You			
	Don't	(15,0)	(5,5)

In the absence of a repeated game, you both live in a pigsty.

In a repeated situation, the *grim trigger strategy*, which says "if you don't share the cleaning, I'll never clean again", can induce both to clean.

One other common method of obtaining a *cooperative solution* is the *tit for tat* strategy.

The *tit for tat* strategy says "If you don't clean today, I won't clean tomorrow". That is, each does what the other did yesterday.

Both the *tit for tat* and the *grim trigger* strategies punish noncooperative behavior with the nash equilibrium.

The Two Firm Pricing Game

Consider a duopoly, two firms. Suppose these firms are choosing between high prices p_H and low prices p_L. Here are the profits of the firms:

		Firm 2	
		p_H	p_L
Firm 1	p_H	(15,15)	(2,25)
	p_L	(25,2)	(5,5)

In a repeated situation, firm 1 might employ the *grim trigger strategy*, which says "if I ever see you price at p_L, I will price at p_L forever after.

This can be used to deter price cutting.

What is the value of cheating to firm 2?

If he doesn't cheat, he gets $15 every time period.

If he cheats, he gets $25 today, but $2 forever after.

Thus cheating provides an immediate gain of $10, and then a loss of $17 per period for the rest of time. So long as each firm expects to be around in the future, and doesn't *discount* the future too highly, it won't want to cheat, and the firms can maintain the high prices. This is called *implicit collusion*. Obtaining the profit-maximizing solution in this way is called a *cooperative equilibrium.*

Two cooperation strategies

Tit for Tat: Copy other's behavior
Grim Trigger: Punish forever if other misbehaves

Advantages
Tit for Tat: can recover from mistakes
Grim Trigger: Maximal deterrence

Best cooperation strategy is a combination of the two: punish for a fixed number of periods.

In the real world, it is often hard to observe whether the other firm cheated.

But there is a simple strategy for producing the cooperative solution that we observe frequently: *meet or beat prices*.