Oligopoly

Cournot

- n firms, each maximizing

$$\pi_i = p(Q)q_i - c_i(q_i)$$

$$0 = \frac{\partial \pi_i}{\partial q_i} = p(Q) + p'(Q)q_i - c_i'(q_i)$$

Implications of FOCs

- Shares $s_i = \frac{q_i}{Q}$

$$\frac{p(Q) - c_i'(q_i)}{p(Q)} = -\frac{p'(Q)q_i}{p(Q)} = -\frac{Qp'(Q)}{p(Q)}\frac{q_i}{Q} = \frac{s_i}{\varepsilon}.$$

- Hirschman Herfindahl Index $HHI = \sum_{i=1}^{n} s_i^2$

$$\sum_{i=1}^{n} \frac{p(Q) - c_i'(q_i)}{p(Q)} s_i = \frac{1}{\varepsilon}\sum_{i=1}^{n} s_i^2 = \frac{HHI}{\varepsilon}$$

Cournot

- Represents a tractable intermediate case between monopoly and competition
- Fixed costs can be used to generate the number of firms
- Widely used in antitrust analysis

Price Dispersion

- Failure of law of one price
- Groceries, airline tickets, etc. have high variation
- Theory attributes to informational or search differences

Theory

- Each consumer buys one unit
- Max price p_m
- s are shoppers, seek best price
- 1-s buy from one of n firms (loyal)
- MC = c
- No pure strategy equilibrium exists
  - either beat low price firm or charge 1

Mixed Strategy Equilibrium

- Seek distribution F of prices
- Given others randomize over F, profits are

$$\pi(p) = (p-c)\left(\frac{1-s}{n} + s(1-F(p))^{n-1}\right)$$

- Profits are constant to induce randomization

$$\pi(p) = (p-c)\left(\frac{1-s}{n} + s(1-F(p))^{n-1}\right) = (p_m - c)\frac{1-s}{n}$$

Solution

- Firm randomizes with distribution F over the interval $[L, p_m]$ and

$$F(p) = \left(1 - \frac{(p_m - p)(1-s)}{s(p-c)n}\right)^{\frac{1}{n-1}}$$

$$L = c + \frac{(p_m - c)\frac{1-s}{n}}{\frac{1-s}{n} + s}$$

I

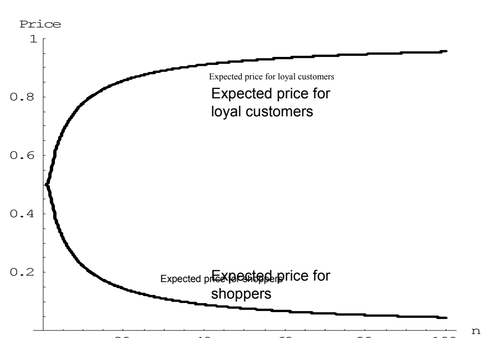
$$n\pi(p) = (p_m - c)(1-s).$$

•Profits Depend on number of non-shoppers

Industry Performance

- Profits Depend on number of non-shoppers $n\pi(p) = (p_m - c)(1-s).$
- Shoppers convey a positive externality to the non-shoppers
- Non-shoppers convey a negative externality on shoppers

Prices



External Effect of Shoppers

Predictions

- Unpredictable prices
  - Grocery prices vary week to week
  - 50% price changes common
- Closed form for price distribution
  - Readily tested
- Negative correlation over time
  - Low prices build up consumer inventories
  - High consumer inventories induce high prices

Hotelling Line

•Products are viewed as located on a line

•Same line represents preferences of consumers

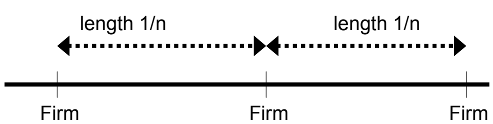
Customer Age

Target Age of Audience

Types

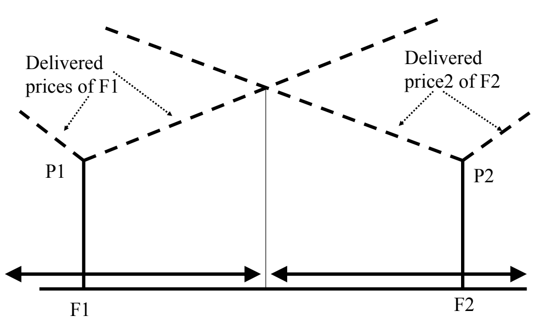
- Quality ("vertical differentiation")
  - gas mileage
  - reliability
  - durability
- Variety ("horizontal differentiation")
  - color
  - style

Circle Model



- 1/n locations on unit circle
- transport cost *t* per unit
- Consumers minimize cost of purchase plus transport

Determination of Prices



Equilibrium Price

- Suppose all firms but one charge p^*
- Remaining firm charges r.
- Equilibrium if profit-maximizing $r = p$.
- Consumer who is indifferent is distance x^*
- $r + tx^* = p + t(1/n - x^*)$

$$x^* = \frac{p + t/n - r}{2t} = \frac{1}{2n} + \frac{p - r}{2t}$$

Profit Maximization

- Profit is $(r - c)2x^* = (r - c)\left(\frac{1}{n} + \frac{p - r}{t}\right)$
- Maximized at

$$0 = \frac{\partial}{\partial r}(r - c)\left(\frac{1}{n} + \frac{p - r}{t}\right) = \left(\frac{1}{n} + \frac{p - r}{t}\right) - \frac{r - c}{t}.$$

- Equilibrium $p = r \Rightarrow p = c + \frac{t}{n}$.

Profits and Efficiency

- Profits are $\frac{t}{n^2}$
- Entry cost F yields $n = \sqrt{t/F}$.
- Total cost of delivery is $\frac{t}{4n} + nF$
- Efficient number of firms is $n^* = \frac{1}{2}\sqrt{t/F}$
- Too many firms!

Agency Theory

- Firm sets commission s, salary y.
- Agent obtains

$$u = sx + y - \frac{x^2}{2a} - s\lambda\sigma^2$$

- Where x is the effort in output units, $1/a$ measures the disutility of effort, σ^2 is the risk, and λ is the risk premium.

Agent Maximization

- A working agent maximizes u over effort x, which yields $x = sa$.
- Increasing shares increase effort.
- Salary y is set to insure the agent accepts the job (u_0 is the reservation utility level):

$$u_0 = s^2a + y - \frac{(sa)^2}{2a} - s\lambda\sigma^2 = y + \tfrac{1}{2}s^2a - s\lambda\sigma^2$$

Salary Determination

- This gives: $y = u_0 - \tfrac{1}{2}s^2a + s\lambda\sigma^2$
- The salary must be higher to compensate for increased risk.

Firm Profits

- The firm earns

$$\pi = (1-s)x - y$$
$$= (1-s)sa - (u_0 - \tfrac{1}{2}s^2a + s\lambda\sigma^2)$$
$$= sa - u_0 - \tfrac{1}{2}s^2a - s\lambda\sigma^2$$

- This provides the firm with the output, minus the cost of effort, the cost of the agent, and the cost of risk.

Firm Maximization

- The firm chooses the agent's share *s*

$$s = 1 - \frac{\lambda}{a}\sigma^2$$

- The share increases in the ability $1/a$ of the agent, and decreases in the riskiness or cost of risk.

Selection of Agent

- Agent paid with a combination of salary and commission
- With a fixed salary, more able agents obtain a higher return.
- Thus, offering a higher commission, lower salary will attract more able agents.
- RE/MAX
- Incentives aren't just about effort, but about agent selection as well