Game Theory

Prisoner's Dilemma

		Column	
Row		Confess	Don't
	Confess	(-10,-10)	(0,-20)
	Don't	(-20,0)	(-1,-1)

Dominant Strategy

- Dominant strategy is best for a player no matter what others do

		Column	
Row		Confess	Don't
	Confess	(-10,-10)	(0,-20)
	Don't	(-20,0)	(-1,-1)

Dominance Solvability

- Iterated elimination of dominated strategies

		Piuny	
MS		Enter	Don't
	Enter	(2,-2)	(5,0)
	Don't	(0,5)	(0,0)

Dominance Solvability

- Iterated elimination of dominated strategies

		Piuny	
MS		Enter	Don't
	Enter	(2,-2)	(5,0)
	Don't	(0,5)	(0,0)

Dominance Solvability

- Iterated elimination of dominated strategies

		Piuny	
MS		Enter	Don't
	Enter	(2,-2)	(5,0)
	Don't	(0,5)	(0,0)

Nash Equilibrium

- Outcome is a Nash equilibrium if it is the result of strategies such that each player's strategy maximizes that player's profits given the strategies of others

Battle of the Sexes

		Woman	
Man		Baseball	Ballet
	Baseball	(3,2)	(1,1)
	Ballet	(0,0)	(2,3)

Matching Pennies

		Column	
Row		Heads	Tails
	Heads	(1,-1)	(-1,1)
	Tails	(-1,1)	(1,-1)

Mixed Strategies

- Column plays H with probability p

		Column		Row E payoff
Row		Heads	Tails	
	Heads	(1,-1)	(-1,1)	$1p + -1(1-p) = 2p-1$
	Tails	(-1,1)	(1,-1)	$-1p + 1(1-p) = 1-2p$

Mixed Strategy Nash Equilibrium

- Strategy for a player is probabilities over the actions
- Those probabilities maximize expected profits
  - So profits from each action chosen with positive probability is the same
  - Indifferent to specific probability values!
- Not mixed = "pure strategy"

Battle of the Sexes

		Woman		
Man		Baseball (prob p)	Ballet (prob $1-p$)	Man's Expected Payoff
	Baseball (prob q)	(3,2)	(1,1)	$3p + 1(1-p) = 1+2p$
	Ballet (prob $1-q$)	(0,0)	(2,3)	$0p + 2(1-p) = 2-2p$
	Woman's E Payoff	$2q + 0(1-q) = 2q$	$1q + 3(1-q) = 3-2q$	

Chicken

		Column	
Row		Swerve	Don't
	Swerve	(0,0)	(-1,1)
	Don't	(1,-1)	(-4,-4)

Cooperation

		Column	
Row		Clean	Don't
	Clean	(10,10)	(0,15)
	Don't	(15,0)	(2,2)

Driving on the Right

		Column	
Row		Left	Right
	Left	(1,1)	(0,0)
	Right	(0,0)	(1,1)

Location Game

		NYC	
LA		No Concession	Tax Rebate
	No Concession	(30,10)	(10,20)
	Tax Rebate	(20,10)	(20,0)

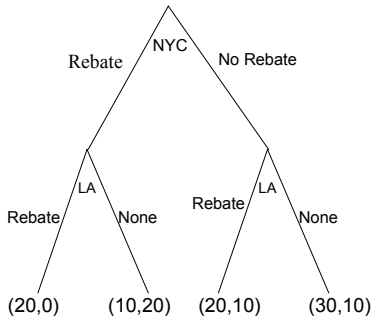
Mudslinging

		Republican	
Dem		Clean	Mud
	Clean	(3,1)	(1,2)
	Mud	(2,1)	(2,0)

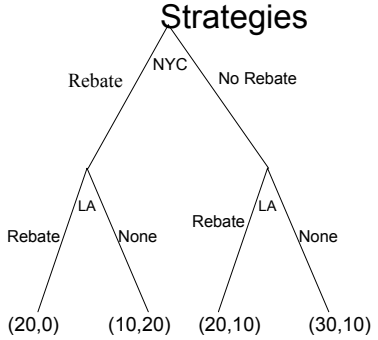
Avoidance

		Rocky	
You		Party 1	Party 2
	Party 1	(5,15)	(20,10)
	Party 2	(15,5)	(0,20)

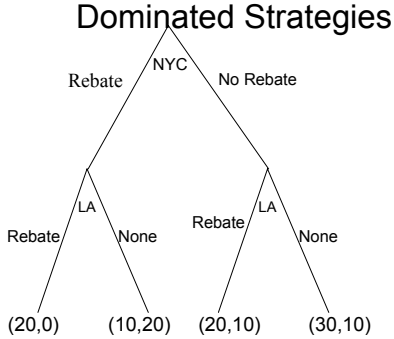
Extensive Form Games



Removing LA's Dominated Strategies



Iterated Elimination of Dominated Strategies



Supergames

- Repeat a given game over and over
- Price cutting game
- Dominant strategy: Low

		Firm 2	
Firm 1		High	Low
	High	(15,15)	(0,25)
	Low	(25,0)	(5,5)

Grim Trigger Strategy

- Cooperate (high price) until rival prices low
  - then price low forever
- If one uses grim trigger, should the other?
- Payoff if defect in time *t*

$$V_t = 15(\delta + \delta^2 + \ldots + \delta^{t-1}) + 25\delta^t + 5(\delta^{t+1} + \delta^{t+2} + \ldots)$$
$$= \frac{15\delta}{1-\delta} - \frac{\delta^t}{1-\delta}(15 - 25(1-\delta) - 5)$$
$$= \frac{15\delta}{1-\delta} - \frac{\delta^t}{1-\delta}(-15 + 25\delta)$$

Cooperation

- If $\delta > 3/5$
- then optimal $t = \infty$, and the best response to the grim trigger strategy is the grim trigger strategy
- Cooperation is an equilibrium in the supergame supported by the grim trigger strategy

Folk Theorem

- An outcome is individually rational for a player if it is as good as the payoff in the worst Nash equilibrium
- Folk theorem: if δ is high enough, any outcome that is individually rational for all players is an equilibrium to the supergame