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Dear Colleagues! May 12/88.

Thanks for the Apraphulian multiplier design. It looks ok but what happens if y=0? In this case, it looks like xy = x. Correct me if I'm wrong! I only have about 2 min to think about each design sent in!

All my best,

A.K. Dewdney

R.P. McAfee & P. Reny
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THE MANDELBROT SET IN MONOCHROME: COMPUTER IMAGE BY YEKTA GURSEL

SCIENTIFIC AMERICAN

COMPUTER RECREATIONS

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Dear Sirs, June 8/88.

Enough said. It works. I guess you know what the next step is: how about building one. It might be fun. If you build 1 multiplier, I might be pursuaded to build some logic. Naturally, we shall have to place a bulk order for lots of small pulleys (avail. to hobbyists, I believe).

Best wishes, Kee Dewdney

R. P. McAfee & P. Reny
Dept. of Economics
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THE MANDELBROT SET IN MONOCHROME: COMPUTER IMAGE BY YEKTA GURSEL

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COMPUTER RECREATIONS

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Professor A.K. Dewdney
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April 21, 1988

Dear Professor Dewdney:

We enclose a description and plan for an analog multiplier, a problem posed in your April, 1988 column in *Scientific American*. We believe the geometry of multiplication described to be the simplest possible (Figure 1). The engineering implementation of this geometry is another matter. Unlike your solutions (except the analog adder), our device exploits the three dimensional nature of the world, and is not a 'flatland' solution. Nevertheless, we hope you like it.

We also note that we designed several multipliers that suffered from the fault that the output was not measured relative to a fixed point in the plane, but instead relative to a rod that moved in the computation, before we noticed that such output could not be inputted to another machine without first shifting the other machines as well - a particularly cumbersome solution. We mention this since it was the error we didn't immediately observe, and hence may also be a problem with other submissions.

This submission is our procedure for corresponding with Sidney Afriat, who is an economic theorist as well.

Sincerely,

R. Preston McAfee

Philip Reny

ANALOG MULTIPLIER

We provide the mathematical development first, and then a description of the Aphraphulian device. The multiplier is based on the geometry of Figure 1. The note on Figure 1 proves the simple geometric relationship that results in the computation of xy, for inputs x and y. The strategy is as follows. Begin with four rigid rods A,B,C, and D, configured as in Figure 2a. Pull rod D up by the amount x, preserving the right angle with rod A (Figure 2b). Now pivot rod B, holding its intersection with A fixed, so that the total distance traveled along C is y (Figure 2c). The distance traveled along D is xy, by the argument in Figure 1.

It remains to implement this description with ropes and other building materials. The rods are rigid: an example would be metal rods. We need two kinds of sliding joints. The first, a universal joint, requires a swivel, and is a circle attached to a half circle by a swivel (see Figures 3a and 3b). These joints are remarkably similar to swivels used to let fishing lures spin, with a bite out of one of the circles. This joint has the property that either rod can slide relative to the other, and the angle at which they meet can vary. However, the two rods can not leave the same "plane" (one is slightly above the other). Thus, in particular, the movement in Figure 2c can be accomplished, as rod B slides along rods C and D, with the angle they meet at changing in the process, but remaining in the same plane.

The second joint needed allows one rod to slide along another while preserving the right angle they meet at. Such a joint is depicted in Figure 3c. This joint is similar to a figure 8 with the bottom o rotated out of the plane of the page.

Both types of joints have little o-rings attached to direct the path of ropes, as necessary.

An illustration of the analog multiplier is given in Figure 4. Several points are worth noting. The rods may be ordered by their distance from the page - coming out of the page. Rod C is the closest to the page, with rod A at the same distance from the page (flush with the page will do). Rod B lies directly above that - to allow it to pass over rod C as rod B swings. Rod D lies above that, as it must also pass over rod C, at least if x is allowed to exceed unity. This is also the reason for the half circles on the swivels - so that as x goes past unity, rod D can pass over rod C without the swivels blocking the movement. The more sophisticated version on the machine has four grooves in rod B and teeth in the ends of the half circles so that there is no "play" in the connections, and no possibility of a rod coming out of the plane parallel to the page. Second, weak springs have been placed on the other ends of the x,y and z ropes, so that the system "zeroes" itself when the tension on the x and y ropes is released. These springs must be weaker than the tension of a pulled rope, perhaps ½ of the typical tension in the system.

Note as well that the machine is also a divider - by pulling on the x and z ropes, one receives the answer $y=x/z$ on the y rope.

One last point is worth noting. If one fixes the value of y, the machine is a scalar multiplier. Having fixed the value of y, if we now distort (bend) the left vertical rod (rod A), one computes some function. The mapping between the shape of the rod A and the function computed is quite complicated, but this provides a strategy for computing a class of functions with simple analog machines.

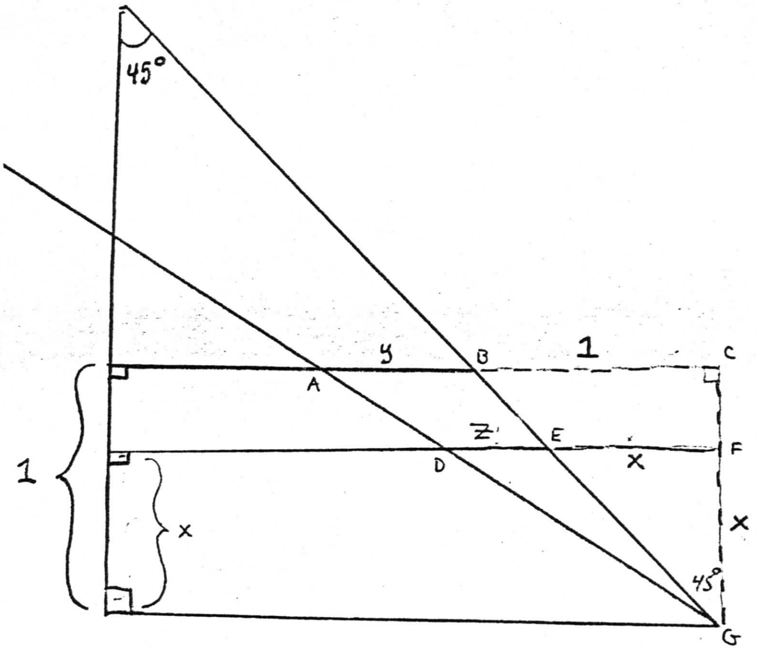


Figure 1: The Basic Geometry: $z = xy$

NOTE: $\Delta ACG \sim \Delta DFG$. Therefore $\frac{|AC|}{|CG|} = \frac{|DF|}{|FG|}$

or $\frac{y+1}{1} = \frac{z+x}{x}$ or $z = xy$, as asserted.

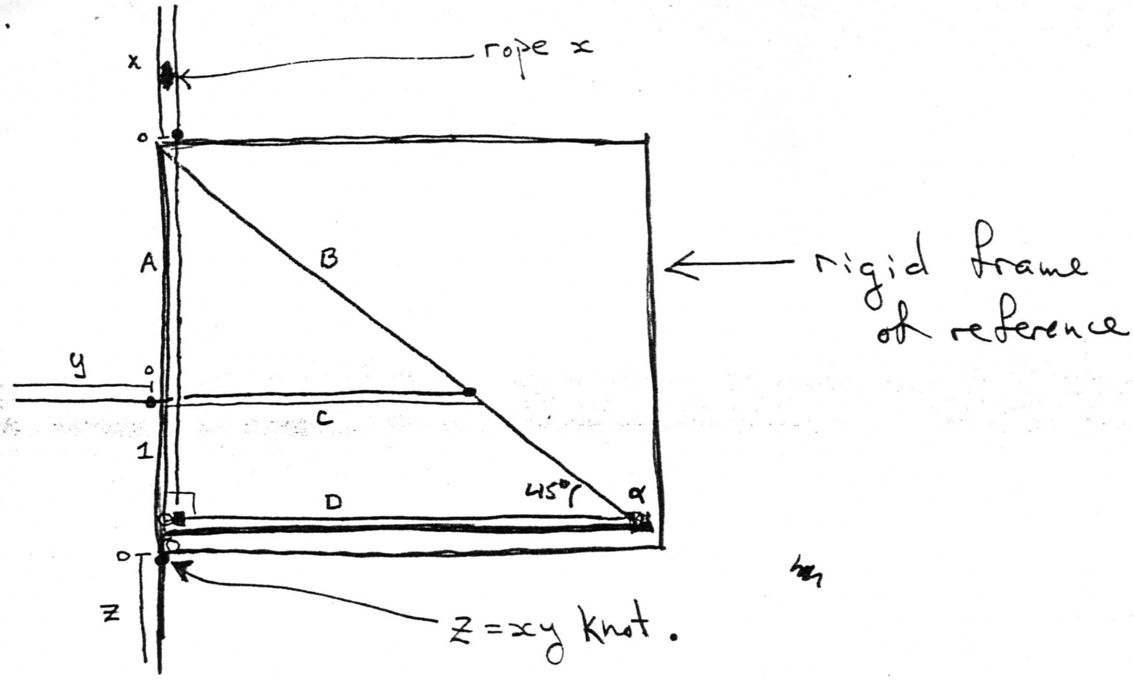


Figure 2a: The initial state: $x = y = z = 0$

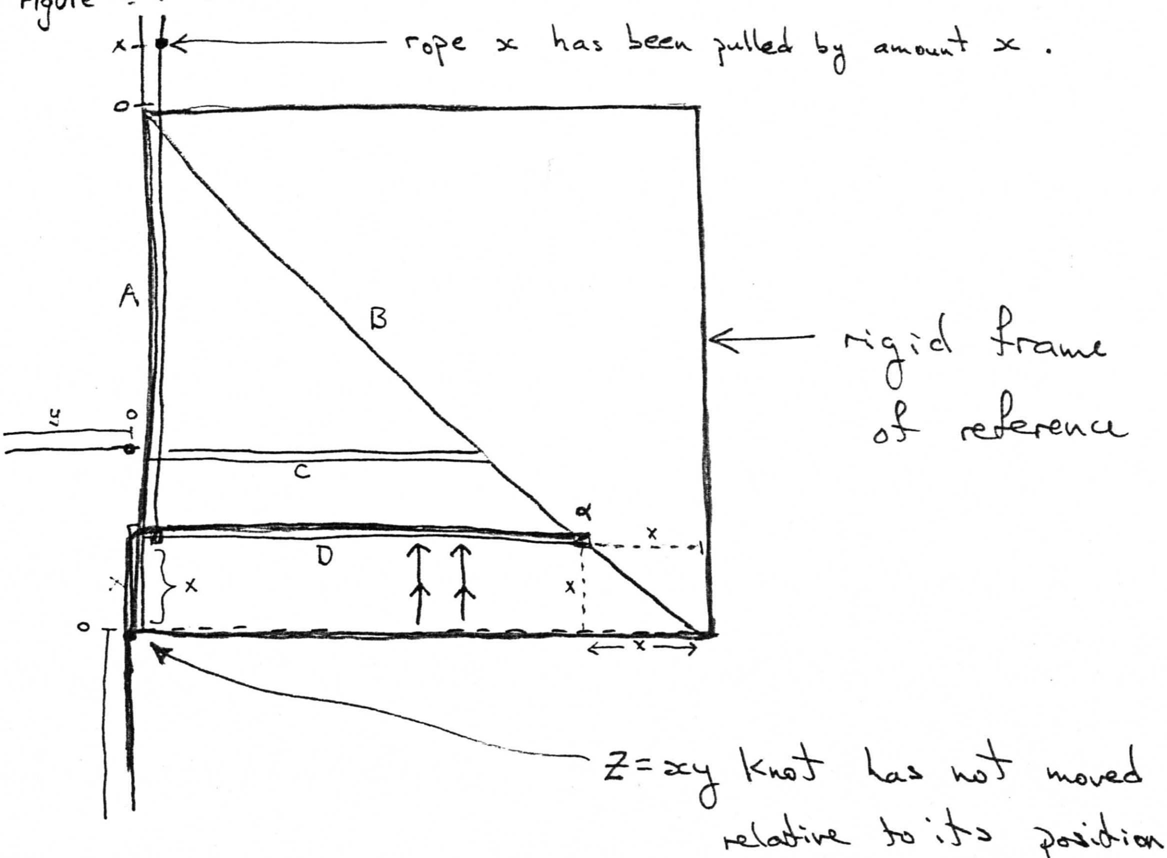


Figure 2b: $x > 0,\ y = z = 0$

$\therefore\ y = 0 \Rightarrow z = xy = 0.$

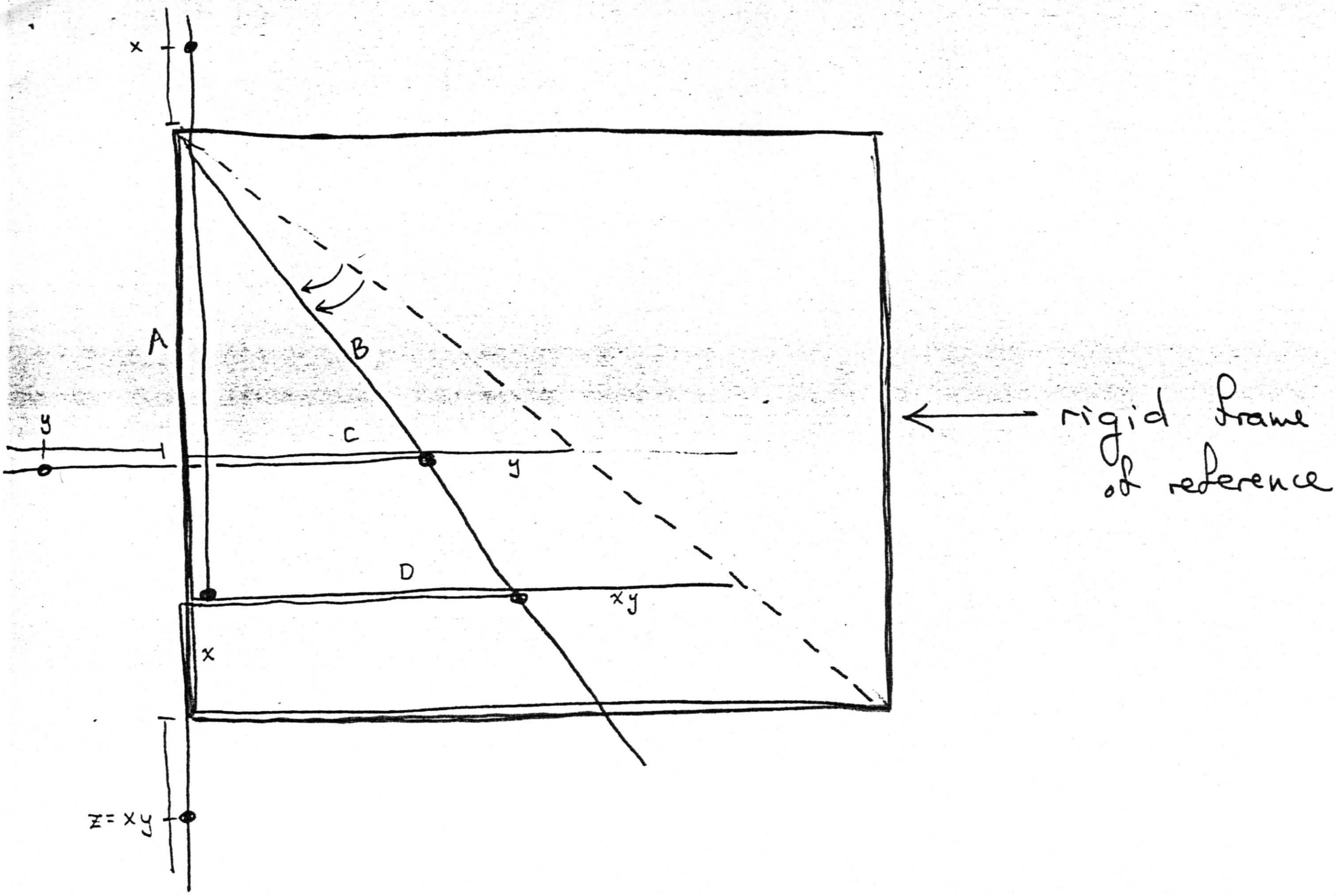


Figure 2c : $x, y > 0$, $z = xy$

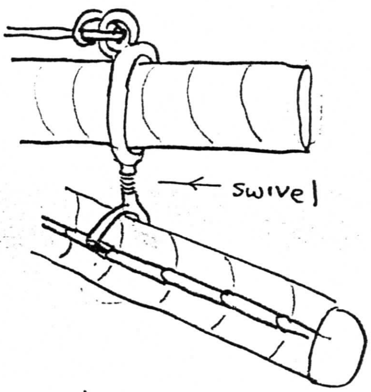


Figure 3a : Note groove that half circle rides in (rod B has two grooves per side)

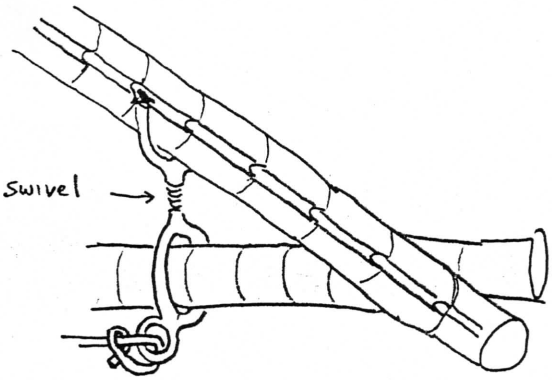


figure 3b : Note Groove

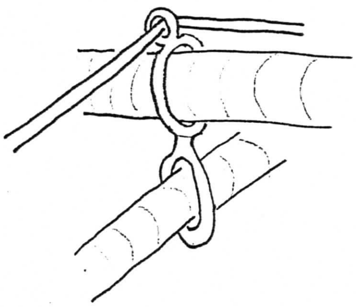


figure 3c

Note : no swivel, but a fixed right angle. Cylinders instead of circles may be used to hold the right angle steady.

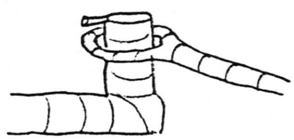
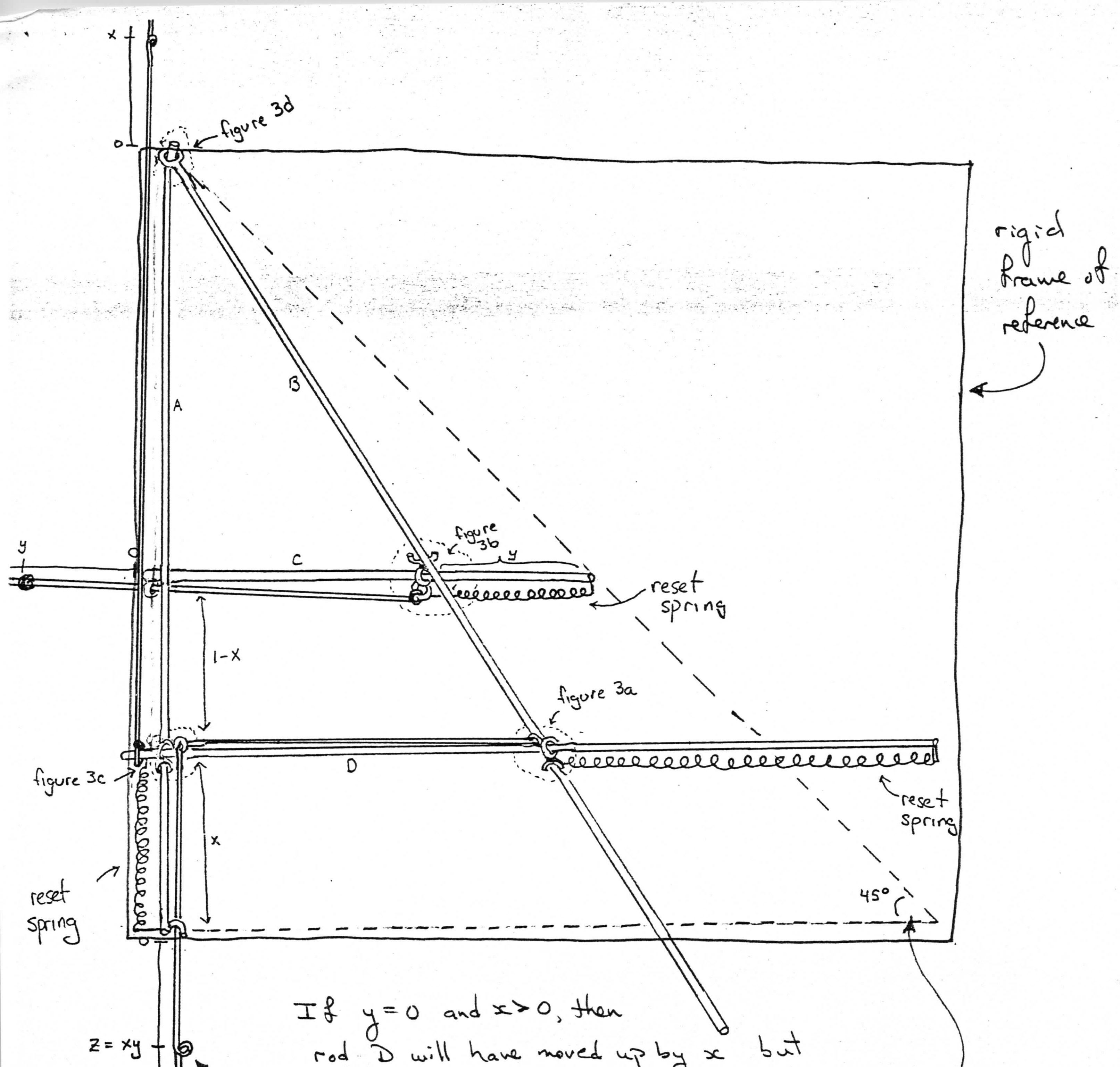


Figure 3d

swing pivot



If $y=0$ and $x>0$, then rod D will have moved up by x but Knot z will <u>not</u> have moved due to 45° angle here.

$\therefore\ xy = 0$ (of course in this figure $y \geq 0$ so that $z \geq 0$; see fig 2b for the case $y=0$)

Figure 4: Analog Multiplier

Pulleys may replace the rope guides to eliminate friction. Neither of us are up to that level of draftsmanship.