

*Journal of
Institutional
and Theoretical
Economics*

JITE

Vol. 172, No. 3

September 2016

Urs Schweizer: Litigated Cases: The Selection Effect Revisited 409–416

Jenny Kragl: Relational Bonus Contracts versus Rank-Order Tournaments with Envious Workers 417–453

Anna E. Shaleva: Culture or Institutions? A Quasi-Experiment on the Origins of Political Trust among Russian Immigrants in Europe 454–474

Till G. Mahr, Eric Nowak, and Roland Rott: The (Ir)relevance of Disclosure of Compliance with Corporate Governance Codes: Empirical Evidence from the German Stock Market 475–520

Nauro F. Campos, Ralitza Dimova, and Ahmad Saleh: Corruption and Economic Growth: An Econometric Survey of the Evidence 521–543

Jiong Gong, R. Preston McAfee, and Michael A. Williams: Fraud Cycles 544–572



Mohr Siebeck

JITE is published 4 times a year by
Mohr Siebeck GmbH & Co. KG, P.O. Box 2040, 72010 Tübingen, Germany

JITE is printed on acid-free and permanent/book paper ∞
meeting all library standards including ANSI/NISO Z39.48-1988.

Papers for Publication

must be in English. They should be submitted online at <https://editorialexpress.com/jite>.
Editorial office: Prof. Dr. Gerd Muehlheusser, Department of Economics, University of
Hamburg, Von-Melle-Park 5, 20146 Hamburg, Germany,
E-mail: jite@mohr.de.

Please follow the Instructions for Authors on the inside back cover.

There is no submission fee. Submission of a paper will be held to imply that it contains original unpublished work and is not being submitted for publication elsewhere.

The Editor does not accept responsibility for damage or loss of paper submitted.
Articles are refereed by specialists.

Books for Review

should be addressed to the Publisher.

Transfer of Rights

When an article is accepted for publication, the exclusive copyright is granted to Mohr Siebeck for publication in a print and an electronic version. Further information on this and the rights retained by the author can be found at www.mohr.de/jite. No one may reproduce or distribute the entire journal or parts of it in a print or an electronic version without the publisher's permission. Please contact rights@mohr.de.

Subscriptions

JITE is published in quarterly issues of approximately 750 pages per year. Back volumes are available on request.

New orders, renewals, claims, and all other correspondence relating to subscriptions should be addressed to the Journals Marketing Manager at the Publisher's address above (Fax +49 7071 51104 or by e-mail to journals@mohr.de). Please quote the name of the journal.

Full Text Online

Free access to the full text online is included in a subscription. We ask institutions with more than 20,000 users to obtain a price quote directly from the publisher. Contact: journals@mohr.de.

In order to set up Online access for institutions / libraries, please go to:

<http://www.ingentaconnect.com/register/institutional>.

In order to set up Online access for private persons, please go to:

<http://www.ingentaconnect.com/register/personal>.

Advertising

For details, contact Tilman Gaebler,
P.O. Box 113, 72403 Bisingen, Germany, Phone +49 7476 3405, Fax +49 7476 3406.

© 2016 Mohr Siebeck GmbH & Co. KG, Tübingen, Germany

This journal and the individual contributions contained in it are protected by the copyright of Mohr Siebeck and may not be reproduced in any form (beyond that permitted by copyright law) without the Publisher's written permission. This applies particularly to reproductions, translations, micro-films as well as storage and processing in electronic systems.

Ask info@mohr.de by e-mail or the Permissions Manager at the publisher's address by mail. – In the U.S.A., permissions may be cleared and payments made through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, Phone (978) 750 8400, Fax (978) 750 47744, www.copyright.com. Other countries may have a local reprographic rights agency for payments.

ISSN 0932-4569

Fraud Cycles

by

Jiong Gong, R. Preston McAfee, and Michael A. Williams*

Received November 8, 2014; in revised form October 15, 2015;
accepted October 15, 2015

Fraud is an ancient crime and one that annually causes hundreds of billions of dollars in losses. We develop an evolutionary theory that suggests cyclical behavior in frauds should be common. We perform a wavelet analysis of the frequencies of fraudulent and nonfraudulent offenses. Our results demonstrate that the frequencies of fraudulent offenses exhibit cyclical behavior that differs markedly from the cyclical behavior of nonfraudulent offenses. (JEL: G38, L2, M49)

1 Introduction

Fraud is as old as civilization itself.¹ Modern frauds include Ponzi and pyramid schemes, securities frauds, corporate accounting financial scandals, medical and automobile insurance frauds, sophisticated art forgeries, the shell game, and the “Nigerian scam,” to name just a few. Fraud is a worldwide crime. In 2009, the Association of Certified Fraud Examiners (“ACFE”) conducted a survey of its approximately 23,000 members and obtained detailed statistics on 1,843 occupational frauds occurring in 106 nations.² The ACFE survey asked its members to estimate the percentage of annual revenue that a typical organization loses to fraud. The median response was five percent, which implies annual global losses of hundreds of billions of dollars. Bernie Madoff’s Ponzi scheme alone, reportedly the

* Jiong Gong (corresponding author): University of International Business and Economics, Beijing, P.R. China; R. Preston McAfee: Microsoft, Redmond (WA), U.S.A.; Michael A. Williams: Competition Economics LLC, Emeryville (CA), U.S.A. We thank Hugo Mialon, Phil Reny, Joel Sobel, Vera te Velde, Thomas Wiseman, and an anonymous referee for helpful comments. We also thank William Havens, David Park, Brijesh Pinto, and Yanfeng Zhu for research assistance.

¹ Aristotle recounts a story of financial fraud in sixth-century B.C. Greece (see Aristotle, 1984).

² The ACFE defines “occupational fraud,” which may occur in public or private organizations, as follows: “The use of one’s occupation for personal enrichment through the deliberate misuse or misapplication of the employing organization’s resources or assets” (Association of Certified Fraud Examiners, 2010, p. 6).

largest in history, caused approximately \$20 billion in losses among its thousand-plus investors (Henriques, 2010). Corporate financial scandals based on fraudulent accounting practices have caused even more economic mayhem. The collapse of Enron in 2001 cost investors and employees over \$70 billion in lost capitalization and retirement benefits (Frontain, 2010).

Public agencies and private firms have taken actions to deter fraud. The responses of organizations to fraud illustrate a general theme: acts of fraud are met with new or modified antifraud measures, which lead to new or modified frauds, which lead to more changes in antifraud measures, and so on. Forensic accountants specializing in fraud detection and deterrence find that the frequency of fraud is a cyclical phenomenon, generating what is called the pendulum swing effect.³ As a leading fraud accounting textbook explains:

“Various pieces of legislation have been passed in response, continuing the cycle of evolving frauds and attempts to control them. [...] The fraud environment can be and often is viewed as a pendulum, swinging from one extreme to the other with little time in between at the proper balancing point. This cycle (pendulum swing) is a natural result of human nature, business cycles, and the nature of legislation and regulation. The cycle can certainly be influenced and controlled to some extent, but it will probably never cease.” (Singleton and Singleton, 2010, pp. 5–7)⁴

Why do frauds come and go? One possible answer is the public’s short memory. The earliest theoretical study of fraud-related phenomena is Lui (1986), who developed a model of corruption deterrence in an overlapping-generation structure. Lui assumes that when corruption becomes more prevalent in the economy, effectively auditing a corrupt official becomes more difficult. This reinforcement leads to variations in government’s effectiveness in deterring corruption, resulting in cyclical patterns of fraud over time. A related topic, the intertemporal variation in business ethics, was studied by Noe and Rebello (1994). They modeled the dynamic interaction between business ethics and economic activities, generating cycling of ethics behavior. Another approach correlates certain types of frauds with the business cycle. For example, corporate financial misrepresentation can be concealed by a boom and revealed by an ensuing bust. Povel, Singh, and Winton (2007) developed

³ Cyclical behavior of prices and outputs has been observed in other economic markets, e.g., the canonical hog cycle. See, e.g., Shonkwiler and Spreen (1986). Agricultural cycles are different in that they have a natural lag created by production processes; such processes are not part of our analysis.

⁴ See also Simić (2005, p. 4): “There are three stages in the [credit-card fraud] cycle. Stage 1 represents familiarity with weaknesses in cards and technology which drives up the value of fraud. Fraud begins to rise as new technologies and new weaknesses are found. Stage 2 represents new solutions implemented to reduce fraud. The solutions are not implemented immediately, and therefore Stage 3 represents time lag for solutions to take effect.” See also Reinstein and Bayou (1998, p. 20): “Fraudsters use many clever schemes to misappropriate company assets and misstate financial statements. Analyzing fraud as a mere historical event can provide an inadequate basis to detect (or prevent) fraud, given its multidimensional, cyclical, and dynamic nature.”

a theoretical model with financial misrepresentation cycles based on investors' vigilance level fluctuating with the boom-and-bust cycle of the economy. All of this work is consistent with the general model we develop. Our approach emphasizes cyclicity created endogenously, rather than driven by an external cycle, but our approach is consistent with such external influences.

Our model is closely related to that of Berentsen and Lengwiler (2004). They used replicator dynamics to develop a doping game that predicts fraud cycles. We build on their insights by making two major extensions. First, we extend their essentially contest setup to a market setup where both demand and supply forces can jointly and endogenously create cyclical behavior. Second, we extend the theory of fraud cycles to a much more general setting where any type of demand and supply interactions can be accommodated, subject to some mild conditions. Our model may also be viewed essentially as a variant of the related inspection-game paper by Berentsen, Bruegger, and Loertscher (2008), where their focus is doping and the role of whistle-blowing. Here, our focus is the interaction of vigilant investors and fraudulent sellers of financial products, where the choice of fraudulent behavior and an investor's purchasing choice are endogenous. Separately, Sutter (2003) provided a theoretical model to study election fraud, where he showed the relevance of both demand and cost factors in the elimination of corrupt election practices. Hyman (2001) analyzed the complexity of relevant parties' differing interests in health care fraud. As a follow-up commentary on Hyman (2001), Feldman (2001) argued that the root cause of fraud in medical programs is distorted high prices coupled with agents' efficiency-seeking activities under price distortion.

Fraud has long been studied in the auditing literature.⁵ Fellingham and Newman (1985) first used a game-theoretic auditing framework to analyze optimal strategies to detect misreporting. Hansen (1993) extended their results to a model of multiple accounts to explore the relationship between auditing strategies and micro characteristics of these accounts, such as the values and distributions of error rates in line items. Caplan (1999) extended this strand of literature in another direction by allowing management to have influence over the internal control system. Corona and Randhawa (2010) considered a two-period setting where the auditor could collude with management in concealing frauds for the sake of his reputation, a scenario in which the auditor can commit fraud himself. Our theory focuses on the cost of auditing combined with imperfect learning. Thus auditors in our approach direct costly resources toward problems but do not learn about what problems are worth investigating instantaneously.

Overall the prior literature focuses primarily on the supply side of frauds, without endogenizing behavior of the demand side (victims). In our approach, we assume an endogenous percentage of businesses are fraudulent while the rest are

⁵ In accounting, fraud means misrepresentation of fact, while misappropriation of assets is termed defalcation. See, e.g., Matsumura and Tucker (1992). We use the word "fraud" in a broader sense to include all acts intended to swindle their victims. Many auditors would argue that they are not detecting fraud but rather detecting misreporting of the underlying transactions.

legitimate, and some buyers on the demand side are vigilant while the rest are not. The interaction of these two forces over time leads to multiple steady states in equilibrium. The reason for multiple steady states is similar to that found in Freeman, Grogger, and Sonstelie (1996). Our main result concerns the convergence to a steady state. We show that cyclic behavior – specifically a spiral – is a robust feature of a large class of evolutionary adaptation models. Applied to the auditing context, our theory is consistent with the pendulum swing effect discussed in the analytical accounting literature.

The intuition for cyclic behavior is that it constitutes a feedback loop, mediated by evolution. When fraud is prevalent, vigilance pays. Increased vigilance reduces the return to fraud, thereby decreasing fraud. The reduction in fraud reduces the return to vigilance, thereby increasing fraud. We make two contributions. First, we show that the predicted outcome (i.e., that the frequency of a given type of fraud will be cyclical) is theoretically robust, and moreover we identify cycles based on the relative response or reaction rates of the two parties – scammers and victims. Interestingly, extremely fast responses by either side tend to eliminate cycles. Moderate adjustment speeds by both sides are necessary, and with a payoff condition sufficient, for cyclic behavior. Second, we empirically examine the frequencies of certain fraudulent and nonfraudulent offenses. Our empirical results corroborate our theoretical predictions.

The remainder of the paper is organized as follows. In section 2 we use the sale of financial products as an example to illustrate a model that generates cyclical behavior. This model can be extended to a general, two-variable model for a variety of management–auditor games where conditions for cyclicity of fraud occurrences are characterized. In section 3 we test two specific predictions of our theory by performing a wavelet analysis of the frequencies of fraudulent and nonfraudulent crimes. Concluding remarks are contained in section 4. All technical proofs are presented in appendix section A.1.

2 *A Model of the Sale of Financial Products*

In this section, we first provide an example where fraudulent behavior displays cyclical patterns, and then generalize it. Suppose wealth management companies in a market offer either legitimate high-quality financial products or scams, and y is the fraction of companies offering high-quality products. All of these products, including scams, are sold as high quality. Potential investors can verify the quality by incurring a verification cost c , and x is the fraction of verifiers. We assume that verifiers never invest in scams. We let v be the net utility of the high-quality product; the net utility of being scammed is assumed to equal zero without loss of generality.

A verifier who encounters a high-quality product, which occurs with probability y , invests, while a verifier who encounters a scam product does not invest but instead searches again, discounting utility due to delay by a rate δ . This means the

verifier obtains a utility in satisfying $u = yv + (1-y)\delta u - c$. The nonverifier obtains utility yv .

We focus on verification and so rule out signaling, introductory prices, reputation, and other solutions studied in the economics literature. These solutions are to some extent consistent with the model. For example, verification could entail checking a firm's reputation. The investor's utility is

$$u = \begin{cases} yv + (1-y)\delta u - c & \text{if verify,} \\ yv & \text{if not,} \end{cases}$$

which simplifies to

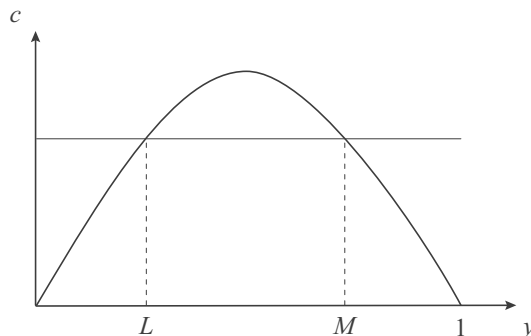
$$u = \begin{cases} \frac{yv - c}{1 - \delta(1-y)} & \text{if verify,} \\ yv & \text{if not.} \end{cases}$$

The net utility gain from verification for an investor equals

$$(1) \quad \frac{yv - c}{1 - \delta(1-y)} - yv = \frac{\delta y(1-y)v - c}{1 - \delta(1-y)}.$$

The existence of at least two equilibria is apparent from (1), because there are typically two levels of y at which the investor is indifferent between verifying and not. In Figure 1, we graph the net utility of verifying as a function of the proportion of law-abiding firms. This utility is zero at both $y = 0$ and $y = 1$, because there is nothing to learn. Thus, if there is a level of y at which the value of verifying exceeds the cost, there will usually be two such levels. Call them L and M . Below L and above M verification does not pay, so the fraction of verifiers will tend to fall.

Figure 1
Net Benefits to Vigilance



The values of L and M can be derived by setting the numerator of (1) to zero, which yields two solutions, $(1/2)(1 \pm \sqrt{1 - 4c/\delta v})$. This means L and M are on the

opposite sides of $1/2$, assuming $c/v < \delta/4$ to guarantee real values for L and M . We maintain this assumption throughout the paper; otherwise the unique solution is zero verification.

A company that sells the high-quality financial product earns a per-unit profit denoted π_g , and any investor who shops with that company purchases the high-quality product. Scammers sell only to nonverifiers, but capture a share λ of the gain in value v of the high-quality product, in addition to the normal per-unit profit π_g .

Verifiers stay in the market longer than nonverifiers, as there are on average $1/y$ searches per verifier, but just one search per nonverifier. Thus, the proportion of nonverifiers per search is $(1-x)/(1-x+x/y)$. The profit of scam companies is $[(1-x)/(1-x+x/y)](\pi_g + \lambda v)$, and the profit of high-quality companies is π_g . The net gain to being a high-quality company, per arriving searcher, is then

$$(2) \quad \pi_g - \frac{1-x}{1-x+x/y}(\pi_g + \lambda v) = \frac{x\pi_g/y - (1-x)\lambda v}{1-x+x/y}.$$

Before further extending our analysis, we briefly introduce a class of market evolution models pioneered by Taylor and Jonker (1978).⁶ Suppose there are n types of interacting market forces indexed by i with market share z_i and utility u_i , which are functions of time. We suppress the time variable for conciseness. The standard replicator dynamics (e.g., Hopkins, 2002; Montgomery, 2010) are given by

$$(3) \quad z'_i = z_i \left(u_i - \sum_{j=1}^n z_j u_j \right).$$

In these models, agents following behavioral strategies that offer utility greater than the average gain market share, while the others lose share. While such models are clearly appropriate for the study of evolution, where utility means “surviving offspring,” they are also reasonable for economic situations where people adapt slowly to changing circumstances. Slow adaptation appears empirically relevant, and indeed might be rational in a larger game where either information or attention has limited availability (e.g., Lucas, 1972).

When there are only two actions, then $z_1 = 1 - z_2$, and equation (3) devolves to $z'_1 = z_1(1 - z_1)(u_1 - u_2)$. Applied to our environment for the financial product quality model, we have two dynamic variables interacting with each other to constitute a system of differential equations. Based on (1) and (2), the two differential

⁶ Evolutionary models first appeared in biology and were later introduced into economics. Our model in this section is similar to the example cited in Friedman (1991, p. 641, footnote 9). For a comprehensive treatment of evolutionary games, see for example Samuelson (1998).

equations are

$$(4) \quad x' = \alpha x(1-x) \frac{\delta y(1-y)v - c}{1 - \delta(1-y)},$$

$$(5) \quad y' = \beta y(1-y) \frac{x\pi_g - (1-x)\lambda v y}{y - xy + x},$$

where α and β are parameters that permit us to vary the speed of adjustment; they correspond to a scaling of the utility of the investors and companies, respectively. There are two steady states in the interior of the unit square as shown in (6) and (7), in addition to (0,0) and (1,1), which are also steady states. Their derivation is presented in appendix section A.1:

$$(6) \quad x^* = 1 - \pi_g \left/ \left[\frac{1}{2} \left(1 \pm \sqrt{1 - \frac{4c}{\delta v}} \right) \lambda v + \pi_g \right] \right.,$$

$$(7) \quad y^* = \frac{1}{2} \left(1 \pm \sqrt{1 - \frac{4c}{\delta v}} \right).$$

Provided $1 > 4c/\delta v$ as previously assumed, there are three values of y consistent with a constant value of x , and moreover x is increasing only in the interval $([1 - \sqrt{1 - 4c/\delta v}]/2, [1 + \sqrt{1 - 4c/\delta v}]/2)$. The intuition is that, if y is large enough, there is no point in verifying, since few companies sell scam products. On the other hand, if y is small, it is too costly to verify, since the expected cost of verifying is c/y . The steady-state equilibrium defined above has two nonzero solutions, one with $y^* > 1/2$, and one with $y^* < 1/2$. The lower value of y^* is unstable, while the higher value is a stable spiral. (There is also a steady state where no customer verifies and all companies are scams.)

PROPOSITION 1 *The higher-value steady state in (6) and (7) constitutes a stable spiral when the following condition is satisfied:*

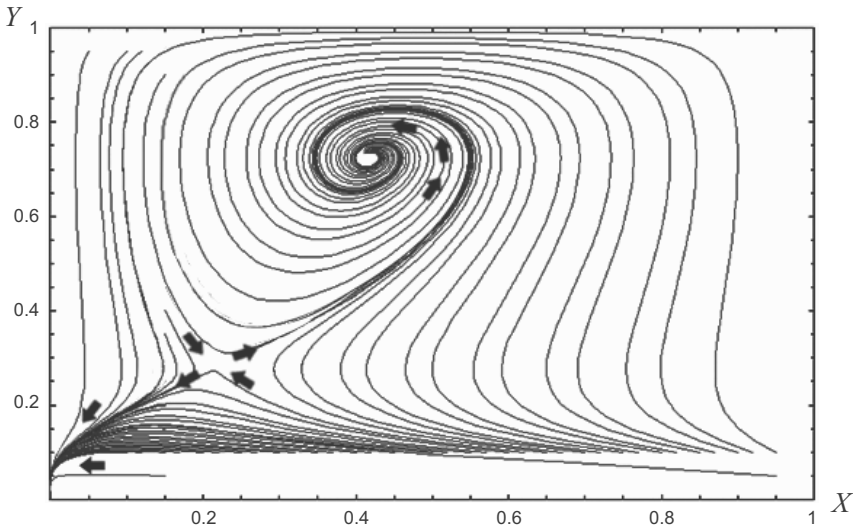
$$(8) \quad \frac{c}{v} < \frac{1}{4}\delta - \frac{1}{4}\delta \left(\frac{2-\delta}{2+8\delta\alpha/\beta-2\delta} \right)^2.$$

All proofs are in appendix section A.1. We used Mathematica to simulate the solution for the system comprising (4) and (5) with schematic values for the parameters of the model, and we plot the results in Figure 2. Of the three steady states shown in the figure, the steady state producing the highest utility is locally stable and is a spiral.

A spiral in the phase plane regulates the behavior of the state variables in the model, x and y , as they converge to a steady state, as shown at the highest convergence point in Figure 2. Along the time dimension, x and y fluctuate or oscillate, forming a periodic convergence pattern. Thus, according to Proposition 1, our model demonstrates the trait of periodic convergence or cyclicity to a stable steady state under external shocks. The cyclicity is caused by market players' nonreinforcing responses to external shocks under stable dynamic systems. As

scammers become more successful, investors react with increased wariness, reducing the profits to scamming. On the other hand, as the wave of scams retreats due to increased investor wariness, investors will gradually become less vigilant, which will in turn cause a new round of scams. As a given type of fraudulent activity becomes more successful and more frequent, firms find it profitable to incur the costs to implement or modify internal controls to detect and deter the activity, thereby reducing the profits to engaging in it. On the other hand, as the success and frequency of a given type of fraud decrease due to the implementation or modification of internal controls, firms will gradually become less vigilant, which will in turn increase the profits to fraudulent activities. This cycle repeats, slowly dampening, until eventually reaching the steady state.

Figure 2
Product Quality Model Phase Diagram



The $(0,0)$ steady state in the figure is locally stable and is always a node. It cannot be cyclic, for the simple reason that market shares cannot be negative. The lower interior steady state is a saddle. It is unstable in the sense that unless the shares line up on one of the arms pointing inward, it cannot be reached, and these arms are a set of measure zero in the space of shares. Unlike common dynamic models where one of the variables is a price or shadow price that can make a discrete jump, both variables in the present model evolve via their equations of motion (4) and (5). Thus, in the present model, saddle stable solutions are very unlikely to be observed.

The condition (8) can be compared with our original assumption that guarantees interior steady states, $c/v < \delta/4$. Consider the case of no speedup where $\alpha = \beta = 1$.

In that case, (8) becomes

$$\frac{c}{v} < \frac{1}{4}\delta - \frac{1}{4}\delta \left(\frac{2-\delta}{2+6\delta} \right)^2.$$

The right-hand side of this inequality is always positive, so that the condition is satisfied for small values of c/v . Moreover, in a sense (8) is not “much” stronger in realistic settings, as it reduces the original upper bound of $\delta/4$ by $(2-\delta)^2/(2+6\delta)^2$. This translates into a c/v upper-bound reduction of less than 5% for δ above $2/3$. The condition (8) is easier to satisfy, as investors react more rapidly and harder to satisfy as the scammers react more rapidly.

The financial-service model can be extended to a much more general theory, using the management–auditor game for illustration, albeit abstracting from its many contextual details. Let $x(t)$ be the probability of an auditor being vigilant, where t is time. Let $y(t)$ denote the probability of the management being honest. Equivalently one can think of $1 - y(t)$ as the probability that an auditor encounters a manager who intentionally misrepresents information for personal financial gain. Such misrepresentation may be legal, where firms mislead rather than lie. Under a dynamic framework where this continuous game is repeatedly played over time, $x(t)$ and $y(t)$ certainly interact with each other. We model the interaction with the system of differential equations in (9). We show that cyclicity would still be observed under mild conditions, no matter what functional forms these two functions take:

$$(9) \quad x'(t) = \alpha f(x(t), y(t)), \quad y'(t) = \beta g(x(t), y(t)).$$

As before, α and β in (9) are parameters to vary the speed of adjustment. This model is general enough to cover a variety of other fraud situations in addition to the management–auditor scenario. The variables $x(t)$ and $1 - y(t)$ also can denote the percentage of government auditors and corrupt officials, respectively, in which case the same model can be used to study corruption. We are interested in when behavior near a stable solution is a spiral, in which case the convergence path displays a cyclical pattern. Mathematically both stability and cyclicity are determined by the trace and determinant of the Jacobian matrix of the system of differential equations (Luenberger, 1979). Let x^* and y^* be a steady state, meaning that they satisfy the following:⁷

$$(10) \quad 0 = f(x^*, y^*), \quad 0 = g(x^*, y^*).$$

Applying Taylor’s expansion around (10) above and ignoring higher-order terms, (9) can be linearized as

$$(11) \quad \begin{bmatrix} x' \\ y' \end{bmatrix} \approx A \begin{bmatrix} x - x^* \\ y - y^* \end{bmatrix}, \quad \text{where } A = \begin{bmatrix} \alpha f_x & \alpha f_y \\ \beta g_x & \beta g_y \end{bmatrix}$$

⁷ We suppress the notation for the time variable t for simplicity, whenever the practice does not cause confusion.

and

$$f_x = \frac{\partial f(x^*, y^*)}{\partial x}, \quad f_y = \frac{\partial f(x^*, y^*)}{\partial y}, \quad g_x = \frac{\partial g(x^*, y^*)}{\partial x}, \quad \text{and} \quad g_y = \frac{\partial g(x^*, y^*)}{\partial y}.$$

Then we have the following proposition:

PROPOSITION 2 *Suppose $\det(A) \neq 0$ and $g_y \neq 0$. A stable steady state is always a sink if $f_y g_x > 0$. If $f_y g_x < 0$, there exists an interval in β/α for which a stable steady state is a spiral. The interval is bounded away from 0 and ∞ .*

If the profits of the two parties react to each other in the same direction, then any stable steady state is a sink. This is implausible in the fraud setting – an increase in the number of honest firms reduces the gains to vigilant auditing, while an increase in vigilant auditing increases the gain to honesty. Thus, typically a fraud model will imply the second case, where $f_y g_x < 0$. Proposition 2 implies that if α and β are very different, in other words, if one type of agent reacts rapidly and the other slowly, then the steady state will be a sink. When they are within intermediate ranges, a spiral results, and a spiral is always possible for some rates of adjustment. Thus spirals are a robust outcome for dynamic evolutionary fraud models.

3 Wavelet Analysis of the Frequencies of Fraudulent and Nonfraudulent Crimes

3.1 FBI Crime Data

In this section we test two specific predictions of the theory by performing a wavelet analysis of the frequencies of fraudulent and nonfraudulent crimes. We use data on fraudulent and nonfraudulent crimes reported by the Federal Bureau of Investigation's National Incident-Based Reporting System (NIBRS) (U.S. Department of Justice, Federal Bureau of Investigation, and Criminal Justice Information Services Division, 2000). Local law enforcement agencies collect data on individual incidents, and they are reported in NIBRS. We obtained NIBRS data for the years 1991–2011. As of 2011, 37 states and 7,618 law enforcement agencies reported NIBRS data.

NIBRS contains data on each incident for 46 offenses.⁸ Frauds reported in NIBRS are counterfeiting/forgery, credit card/automatic teller machine (ATM) fraud, embezzlement, false pretenses/swindle/confidence game, impersonation, welfare fraud, and wire fraud. Specific facts for each incident are reported, such as the date, location type, method of entry, offense attempted/completed, offender age, offender sex, offender race, and type of offense committed.⁹ A given incident may result in more than one offense being reported. For example, if an incident

⁸ For the list of 46 offenses reported, see U.S. Department of Justice, Federal Bureau of Investigation, and Criminal Justice Information Services Division (2000, pp. 10–11).

⁹ For the list of facts reported for each incident, see U.S. Department of Justice, Federal Bureau of Investigation, and Criminal Justice Information Services Division (2000, pp. 6–7).

involves impersonation and assault, these two offenses will be reported separately. NIBRS data are more detailed, accurate, and meaningful than traditional summary crime reporting data, such as data reported in the Uniform Crime Reporting Handbook.

Since 1991, the number of local law enforcement agencies reporting data to NIBRS has increased substantially (see Table A1 in appendix section A.2). In order to hold constant the set of enforcement agencies for a given offense, we use the set of enforcement agencies that reported at least one incident for that offense in 2000. We then use all the incidents from that set of enforcement agencies for the given offense over the period January 2000 through December 2011.¹⁰ Table A2 in appendix section A.2 shows the annual number of incidents for each offense.

3.2 Wavelet Analysis

We test two predictions of the theory regarding the frequencies of different types of frauds. First, do the frequencies of fraudulent crimes display different cyclical characteristics from those of nonfraudulent crimes such as assault, burglary/breaking and entering, larceny/theft, motor vehicle theft, and robbery? The frequencies of these and other violent crimes are known to follow annual cycles (as affected in large part by holidays and weather; see, e.g., McDowall, Loftin, and Pate, 2012) and longer business cycles (see, e.g., Raphael and Winter-Ebmer, 2001). In contrast, our theory predicts that the frequencies of fraudulent crimes exhibit cyclical behavior uncorrelated with seasonal conditions or longer macroeconomic fluctuations in unemployment rates or income levels. Second, are the frequencies of different types of fraudulent crimes cyclical, or are they instead consistent with random behavior? Our theory predicts cyclical behavior in the frequencies of fraudulent crimes based on the feedback between fraud and antifraud measures.

To analyze the NIBRS data, we use wavelet analysis (Percival and Walden, 2000; Crowley, 2007). Wavelets are a mathematical tool that can be used to analyze time-series data. A desirable feature of wavelet analysis is that the time-series data do not need to be stationary. Wavelet analysis transforms a time series into several time series, each reflecting a different time scale. Wavelet analysis has been described as “like sliding magnifier glasses with different power over the time series to see different levels of detail” (Krüger, 2011, p. 180). The highest scale transforms the time series with a focus on the whole picture and less on the details. Conversely, the lowest scale transforms the time series with a focus on the details and less on

¹⁰ Our results are not sensitive to this restriction on the set of local law enforcement agencies. Our basic finding (i.e., that there exists a marked difference in the cyclical behavior of the frequencies of fraudulent and nonfraudulent offenses) holds using NIBRS data for 1991–2011, with the requirement that we use only those local law enforcement agencies that, by offense, reported at least one incident in 1991. The advantage of restricting the period to 2000–2011 is that we can use data from many more local law enforcement agencies in constructing a consistent time-series data set for each offense. In particular, using the 1991–2011 period, rather than 2000–2011, causes a 77.6% to 99.5% reduction in the annual number of incidents, depending on the offense.

the whole picture. Wavelet analysis is useful in observing time-series data across a spectrum of temporal scales.

We perform a wavelet analysis using the NIBRS data for fraudulent and non-fraudulent offenses. Specifically, we use the maximum-overlap discrete wavelet transform (MODWT) to analyze the NIBRS data (Constantine and Percival, 2010). For the MODWT, we use the least-asymmetric wavelet filter of length eight, with periodic boundary conditions (Percival and Walden, 2000). We use four scale levels. The first scale represents a 2–4-month period; the second scale a 4–8-month period; the third scale an 8–16-month period; and the fourth scale a 16–32-month period.

We first perform a wavelet analysis of selected nonfraudulent offenses. We use MODWT to analyze cyclical behavior in NIBRS data for assault, burglary/breaking and entering, larceny/theft, motor vehicle theft, and robbery. Figure A1 (in appendix section A.2) shows the MODWT decomposition analysis for assault. The vertical axis shows the scales (d1–d4) for each graph and the smooth (s4). The wavelet scales capture the behavior of transient and oscillatory frequencies at different time scales, while the smooth represents a moving average of the time-series data. The horizontal axis represents the number of months between January 2000 and December 2011. The MODWT analysis shows that the frequency of assault has the most energy in the scale d3, which represents an 8–16-month cycle. (The *energy* of a scale is the squared norm of the wavelet coefficient vector.) This result indicates that the frequency of assault follows an annual cycle.

This finding is made clear in the accompanying variance decomposition by scale for assault, shown in Figure A2.¹¹ The vertical axis represents the sum of all of the discrete wavelet transform coefficients squared for each scale, and the horizontal axis represents each scale (d1–d4). As the histogram shows, scale d3 accounts for 64 percent of the variance in the monthly frequency of assault. Thus, the frequency of assault has an annual cycle.

Our MODWT analyses for the nonfraudulent crimes of burglary/breaking and entering, larceny/theft, motor vehicle theft, and robbery are shown in Figures A3–A10. The results show that scale d3 consistently accounts for most of the energy in the MODWT decompositions and most of the variance in the variance decompositions. Across all five of the nonfraudulent offenses, scale d3 accounts for 48–72 percent of the variance in the monthly frequencies of these offenses (Table 1). We conclude that the frequencies of these nonfraudulent offenses have an annual cycle.

We next perform a wavelet analysis of fraudulent offenses. We use MODWT to analyze the frequency of incidents in the NIBRS data for fraudulent offenses. Figures A11–A24 show the MODWT decomposition and variance decomposition analyses, with the variance decomposition results summarized in Table 1. The results show that the scales with the most energy are d1 (counterfeiting/forgery),

¹¹ We thank Patrick M. Crowley for providing us with the variance-decomposition R code.

Table 1
 Variance Decomposition of Fraudulent and Nonfraudulent Offenses (percentages;
italic entries indicate maximum-variance decomposition per offense)

Offense	Wavelet scale			
	d1 (2–4 months)	d2 (4–8 months)	d3 (8–16 months)	d4 (16–32 months)
<i>Nonfraudulent</i>				
Assault ¹	16.5	13.5	<i>63.9</i>	6.1
Burglary	12.5	14.3	<i>65.8</i>	7.4
Larceny/Theft	11.3	11.3	<i>71.5</i>	6.0
Motor vehicle theft	15.9	15.8	<i>57.6</i>	10.7
Robbery	22.1	23.4	<i>48.3</i>	6.2
<i>Fraudulent</i>				
Counterfeiting/Forgery	<i>39.3</i>	22.7	25.4	12.7
Credit card/ATM fraud	18.8	15.3	24.7	<i>41.2</i>
Embezzlement	26.0	28.2	<i>32.7</i>	13.1
False pretenses/Swindle/ Confidence game	32.7	13.2	20.5	<i>33.7</i>
Impersonation	21.6	16.7	23.5	<i>38.2</i>
Welfare fraud	<i>55.3</i>	24.5	10.4	9.8
Wire fraud	24.9	20.8	21.1	<i>33.2</i>

d4 (credit card/ATM fraud), d3 (embezzlement), d4 (false pretenses/swindle/confidence game), d4 (impersonation), d1 (welfare fraud), and d4 (wire fraud).

The wavelet results for nonfraudulent offenses differ markedly from those for fraudulent offenses. The frequencies of the nonfraudulent offenses exhibit strong annual cycles. In contrast, the frequencies of the fraudulent offenses do not exhibit strong annual cycles. For four of the fraudulent offenses (credit card/ATM fraud, false pretenses/swindle/confidence game, impersonation, and wire fraud), scale d4 (16–32 months) accounts for 33–41 percent of the variance. For counterfeiting/forgery and welfare fraud, d1 (2–4 months) accounts for 39–55 percent of the variance. For embezzlement, d3 (8–16 months) accounts for 33 percent of the variance. However, with just twelve years of monthly data, the wavelet analysis can only fully resolve cycles up to six years with certainty. So we cannot rule out the possibility of longer cycles. If fraudulent offenses do exhibit such longer cycles, those cycles will continue to confirm our basic finding that the frequencies of fraudulent offenses exhibit cyclical behavior that differs markedly from the cyclical behavior of nonfraudulent offenses.

As a robustness test, we also performed our analysis using alternative wavelet functions with periodic boundary conditions: a least-asymmetric wavelet filter of length ten; Daubechies of lengths four, six, and eight; and best localized of lengths 14 and 20. For each of these alternative wavelet functions, the wavelet scale with the maximum-variance decomposition for each of the twelve offenses shown in

appendix section A.2 was the same as that using the least-asymmetric wavelet filter of length eight, with periodic boundary conditions. We also performed our analysis changing the boundary conditions from periodic to reflection. For eight of the twelve offenses shown in appendix section A.2, the wavelet scale with the maximum-variance decomposition was the same using the least-asymmetric wavelet filter of length eight with (1) periodic boundary conditions as with (2) reflection boundary conditions. For four of the offenses (credit card/ATM fraud, false pretenses/swindle/confidence game, impersonation, and wire fraud), the wavelet scale with the maximum-variance decomposition changed from d4 (i.e., a 16–32 month period) to d1 (i.e., a 2–4 month period). Although the wavelet scale with the maximum-variance decomposition changed for these four offenses upon switching from periodic to reflection boundary conditions, the maximum-variance decompositions did not change to the annual scale (i.e., d3 with an 8- to 16-month period) associated with nonfraudulent offenses (see appendix section A.2). In sum, both of these robustness tests confirm our basic finding: the frequencies of fraudulent offenses exhibit cyclical behavior that differs markedly from the cyclical behavior of nonfraudulent offenses.

A further indication of the basic difference in the cyclical behavior of the frequencies of fraudulent and nonfraudulent offenses can be obtained by calculating the relevant wavelet multiple correlations (Fernández-Macho, 2011, 2012). The wavelet multiple correlation measures the overall statistical relationship that exists at different time scales among a set of observations on a multivariate random variable. The wavelet multiple correlation analysis for nonfraudulent offenses is shown in Figure A25. The results show that the wavelet multiple correlation between the nonfraudulent offenses reaches a maximum at scale d3 (8–16 months) with a value of 0.998 and a 95-percent confidence interval of 0.993 to 0.999. This demonstrates that the cycles of the nonfraudulent offenses are almost perfectly correlated at scale d3. The wavelet multiple correlation analysis for fraudulent offenses is shown in Figure A26. The results show that the wavelet multiple correlation between the fraudulent offenses reaches a maximum at scale d4 (16–32 months) with a value of 0.977 and a 95-percent confidence interval of 0.844 to 0.997. This demonstrates that the cycles of the fraudulent offenses have their highest correlation at scale d4, but they exhibit substantial variations in cycles, as indicated by the lower wavelet multiple correlations across the four wavelet scales and the large 95-percent confidence intervals.

Finally, our wavelet results on cycles in the frequencies of fraudulent and nonfraudulent offenses are not likely due to random variation. Figure A27 shows the MODWT decomposition analysis for a random sample of 144 numbers (i.e., the same as the number of months in our MODWT analyses). Figure A28 shows the corresponding variance decomposition. The results show that scale d1 has the most energy, accounting for 52 percent of the variance, with scales d2, d3, and d4 accounting for declining amounts of the variance. None of the nonfraudulent and only one of the fraudulent offenses (welfare fraud) appears close to the wavelet analysis of the random sample of 144 numbers. The MODWT decomposition for

welfare fraud (Figure A21) exhibits similar energies at scales d3 and d4 to the corresponding energies for the random sample (Figure A27). However, the variance decomposition for welfare fraud (Figure A22) shows that the percentages of the variance accounted for by the d3 and d4 wavelet scales are approximately equal, which is inconsistent with the d3 and d4 wavelet scales for the random sample (Figure A28).

To sum up, Table 1 and Figures A12–A24 show that the fraudulent offenses have wavelet cycles inconsistent with random frequencies of incidents, as shown in Figures A27 and A28, with the possible exception of welfare fraud. Thus, the frequencies of incidents for these fraudulent offenses are not random. But are the frequencies of incidents for these offenses cyclical? We know from prior research that the frequencies of nonfraudulent offenses such as assault, burglary/breaking and entering, larceny/theft, motor vehicle theft, and robbery have distinct annual cycles (see, e.g., McDowall, Loftin, and Pate, 2012). The wavelet analysis of the NIBRS data (as shown in Table 1) correctly reveals these annual cycles, since scale d3 (8–16 months) explains most of the variance in the frequencies of these five nonfraudulent offenses.

The wavelet analysis reveals that four fraudulent offenses (credit card/ATM fraud, false pretenses/swindle/confidence game, impersonation, and wire fraud) have most of the energy in the frequencies of their incidents in the d4 scale. Therefore, the frequencies of incidents for these four fraudulent offenses have cycles of 16–32 months. Note that the cycle that explains most of the variance in the frequencies of incidents for these four offenses is the longest cycle used in our study. In principle, the cycles for these four offenses could be longer (but not shorter); however, revealing that fact would require a longer time series than the 144 months used in our wavelet analysis. Similarly, the wavelet analysis reveals that one fraudulent offense (embezzlement) has most of the energy in the frequency of its incidents in the d3 scale. Therefore, the frequency of incidents for embezzlement has a cycle of 8–16 months. Finally, the wavelet analysis reveals that two fraudulent offenses (counterfeiting/forgery and welfare fraud) have most of the energy in the frequencies of their incidents in the d1 scale. Therefore, the frequencies of incidents for these two fraudulent offenses have cycles of 2–4 months. However, a cycle of only 2–4 months appears too short to be consistent with our fraud cycle theory. We suspect that some other factor likely accounts for the observed cyclicity of these two fraudulent offenses, and, as noted, the MODWT decomposition for welfare fraud generally exhibits similar energies to the corresponding energies for the random sample.

4 Concluding Remarks

We have proposed a new theory to show that cyclicity follows inherently from interacting market forces to (1) shun and eliminate frauds on the demand side and (2) sustain and perpetuate frauds on the supply side. Applied to the auditor–

management context, this means that the battle between the two sides displays a pendulum swing effect, and ultimately there will always be some level of frauds in the steady state. We then reviewed historical data on frauds and identified empirical evidence of peak and trough patterns associated with some common frauds. The data suggest that frauds tend to follow a cyclical path of their own that cannot be explained by seasonal, annual, or business-cycle reasons. The strategic interaction between the demand and supply forces of fraud causes the cyclical path to equilibrium to be robust to outside shocks, such as federal and state legislation. We identify conditions under which such cyclical behavior occurs.

The fit of the empirical work to the theory bears some discussion. Some of the reactions to accounting fraud or other types of fraud involve new legislation. In principle, such legislation means that fraud cannot be cyclic, as the rules of the game change and society never returns to a pre-legislation state. We consider, however, that even an evolutionary theory has value and may in fact be more appropriate than a theory based on the law. The quantities that cycle in our model are the number of attempted frauds and the vigilance of the intended victims. While changes in the law may affect the precise nature of attempted fraud, the law is not a substitute for vigilance, and the level of vigilance is probably a more important determinant of fraud than the law. Thus, while our model will not predict the nature of today's fraud, it does offer an account of fraud cycles independent of either business cycles or annual cycles driven by holidays and weather.

Fundamentally, cyclicity is caused by market players' nonreinforcing responses to external shocks under stable dynamic systems. As scammers become more successful, customers react with increased wariness, reducing the return to scamming. This cyclicity is substantially different from the supply-and-demand hog cycle, which was predicated on delayed reactions – hog ranchers increasing the stock in reaction to today's prices, which results in an increase in supply next season. In contrast, the cyclicity discovered in this paper is a consequence of endogenous delay, driven by the evolutionary dynamics. The present theory is general enough to investigate a variety of frauds in addition to that pertaining to the auditor–management game. The characterization of local behavior greatly simplifies the understanding of system equilibrium behavior without actually solving for the solutions, which can be immensely complicated even under a simple specification like that provided in section 2.

Future research effort points in three directions. One is to empirically study more types of frauds to further evaluate the hypothesis of cyclical behavior. Unlike business cycles, for which data are collected systematically by macroeconomic policy authorities and other economic research institutions, fraud cycles are subtle to discern because they arise from illegal behavior. Observation tends to reduce fraud through increased awareness. On the theoretical side, our model abstracts from the detailed fabric of the auditor–management game to a macro-auditing perspective over the long run. It would be interesting to investigate further implications of a micro-auditing setup with more details in the model. Finally, our model primari-

ly focuses on the spiral steady state, which results in cyclical behavior. A natural extension is the case of swindlers choosing from a set of potential frauds.

Appendix

A.1 Derivation of the Steady States

First note from (4) that $x' = 0$ arises when $y^2 - y + c/\delta v = 0$, which, if it has a solution in $[0, 1]$, has two generically. These solutions are $y^* = (1 \pm \sqrt{1 - 4c/\delta v})/2$. By (5), the steady states in y arise when $x\pi_g/y - (1 - x)\lambda v = 0$, or $x^* = \lambda v / (\lambda v + \pi_g/y^*)$. Plug in the solution for y^* to yield the two steady, nonzero states for (x^*, y^*) .

PROOF OF PROPOSITION 1 We first linearize the system by first-order approximation around the steady state:

$$\begin{aligned} \begin{bmatrix} x' \\ y' \end{bmatrix} &\approx \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x - x^* \\ y - y^* \end{bmatrix} \\ &= \begin{bmatrix} \alpha(1 - 2x^*) \frac{\delta y^*(1 - y^*)v - c}{1 - \delta(1 - y^*)} & \alpha x^*(1 - x^*) \frac{\delta(1 - 2y^*)v}{1 - \delta(1 - y^*)} \\ \beta y^*(1 - y^*) \frac{\pi_g/y^* + \lambda v}{1 - x^* + x^*/y^*} & \beta y^*(1 - y^*) \frac{-(1 - x^*)\lambda v/y^*}{1 - x^* + x^*/y^*} \end{bmatrix} \begin{bmatrix} x - x^* \\ y - y^* \end{bmatrix}. \end{aligned}$$

Note that $a_{11} = 0$, because $\delta y^*(1 - y^*)v - c = 0$. It is obvious that every factor in a_{21} is positive, so that $a_{21} > 0$. The same can be said about a_{22} , except that $-(1 - x^*)\lambda v/y^*$ in a_{22} is negative. Therefore $a_{22} < 0$. Finally, $a_{12} < 0$, since $1 - 2y^* < 0$, as the value of y^* of our interest is the higher one, and $1 - \delta(1 - y^*) > 0$. Then the stability result follows from the standard stability condition for steady states (Luenberger, 1979).

To have a steady state be a spiral, one needs the condition $\text{Tr}(A)^2 - 4\det(A) < 0$. We have

$$\begin{aligned} \text{Tr}(A)^2 - 4\det(A) &= \left[\beta y^*(1 - y^*) \frac{-(1 - x^*)\lambda v/y^*}{1 - x^* + x^*/y^*} \right]^2 \\ &\quad + 4\alpha\beta x^*(1 - x^*)y^*(1 - y^*) \left[\frac{\delta(1 - 2y^*)v}{1 - \delta(1 - y^*)} \right] \left[\frac{\pi_g/y^* + \lambda v}{1 - x^* + x^*/y^*} \right] \\ &= \frac{\beta(1 - x^*)y^*(1 - y^*)}{1 - x^* + x^*/y^*} \left\{ \frac{\beta(1 - x^*)(1 - y^*)}{y^*(1 - x^*) + x^*} (\lambda v)^2 \right. \\ &\quad \left. + 4\alpha x^* \frac{\delta(1 - 2y^*)v}{1 - \delta(1 - y^*)} (\lambda v/x^*) \right\} \\ \text{(A1)} \quad &= (\beta\lambda v^2) \frac{(1 - x^*)y^*(1 - y^*)}{1 - x^* + x^*/y^*} \left\{ \frac{\beta(1 - y^*)\lambda}{y^*(1 + \lambda v/\pi_g)} + 4 \frac{\alpha\delta(1 - 2y^*)}{1 - \delta(1 - y^*)} \right\}. \end{aligned}$$

The above steps use the steady-state condition $x^* \pi_g - (1-x^*) \lambda v y^* = 0$. For (A1) to be negative, it suffices to show the third factor in (A1), which is in the large braces, to be negative, since the factors before it are both positive. For convenience denote $\theta = \sqrt{1-4c/(\delta v)}$ so that the high-value steady state $y^* = (1+\theta)/2$, and plug it into the last part of (A1). We then need to show the following:

$$\frac{\beta(1-y^*)\lambda}{y^*(1+\lambda v/\pi_g)} + 4 \frac{\alpha\delta(1-2y^*)}{1-\delta(1-y^*)} = \frac{\beta(1-\theta)\lambda}{(1+\theta)(1+\lambda v/\pi_g)} - \frac{8\alpha\delta\theta}{2-\delta(1-\theta)} < 0.$$

After arranging terms, we have

$$(A2) \quad 2\beta(1-\theta)\lambda - \beta\delta(1-\theta)^2\lambda - 8\alpha\delta\theta(1+\theta) \left(1 + \frac{\lambda v}{\pi_g}\right) < 0.$$

Note that (A2) does not always hold, for example when $\theta \rightarrow 0$. But since $1 + \lambda v/\pi_g > \lambda$, a sufficient condition for (A2) is $2\beta(1-\theta) - \beta\delta(1-\theta)^2 - 8\alpha\delta\theta(1+\theta) < 0$, or

$$(A3) \quad \delta(8\alpha + \beta)\theta^2 + (2\beta + 8\alpha\delta - 2\beta\delta)\theta + \beta(\delta - 2) > 0.$$

Since the first term of (A3) is always positive, we just look at a sufficient condition:

$$(A4) \quad (2\beta + 8\alpha\delta - 2\beta\delta)\theta + \beta(\delta - 2) > 0.$$

Plugging the definition of θ and solving for c/v , it can be shown that (A4) is satisfied under (8). *Q.E.D.*

PROOF OF PROPOSITION 2 The sign of $\text{Tr}(A)^2 - 4\det(A) = (\alpha f_x - \beta g_y)^2 + 4\alpha\beta f_y g_x$ is the same as the sign of $(f_x - \gamma g_y)^2 + 4\gamma f_y g_x$, where $\gamma = \beta/\alpha$. The sign of the latter can be examined by looking at its minimum, since it is convex in γ . Solving for that minimum using the first-order condition yields

$$\gamma^* = \frac{f_x}{g_y} - \frac{2\alpha\beta f_y g_x}{g_y^2}.$$

Plugging it back to the objective function gives

$$\min \text{Tr}(A)^2 - 4\det(A) = 4 \left(\frac{\alpha f_y \beta g_x}{g_y^2} \right) (\alpha f_x \beta g_y - \alpha f_y \beta g_x).$$

Now $g_y^2 > 0$ and $\alpha f_x \beta g_y - \alpha f_y \beta g_x > 0$ by the stability condition. Thus the sign of $\text{Tr}(A)^2 - 4\det(A)$ is determined by the sign of $f_y g_x$ when $\alpha\beta > 0$. *Q.E.D.*

A.2 Additional Tables and Figures

Table A1
Number of Reporting Agencies by Offense and Year: 1991–2011

Offense	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
<i>Nonfraudulent</i>																					
Assault ¹	529	813	1,251	1,307	1,092	1,310	1,717	2,178	2,591	2,747	3,304	3,456	3,817	3,997	4,199	4,351	4,468	4,734	5,137	5,214	5,402
Burglary/Breaking and entering	526	799	1,205	1,190	1,016	1,230	1,625	2,053	2,417	2,563	3,123	3,244	3,603	3,780	4,005	4,120	4,226	4,458	4,856	4,976	5,128
Larceny/Theft	546	829	1,281	1,339	1,140	1,353	1,752	2,232	2,624	2,754	3,372	3,498	3,872	4,087	4,277	4,420	4,527	4,807	5,218	5,297	5,514
Motor vehicle theft	452	726	1,024	1,075	971	1,170	1,491	1,911	2,225	2,369	2,818	2,939	3,232	3,394	3,601	3,717	3,745	3,901	4,159	4,232	4,289
Robbery	348	432	584	572	521	660	813	1,063	1,254	1,329	1,573	1,659	1,828	1,963	2,024	2,171	2,254	2,335	2,549	2,528	2,601
<i>Fraudulent</i>																					
Counterfeiting/ Forgery	239	418	729	783	733	786	1,075	1,471	1,763	1,907	2,371	2,486	2,810	3,052	3,260	3,338	3,362	3,561	3,846	3,848	3,949
Credit card/ ATM fraud	114	177	341	345	379	499	648	907	1,117	1,367	1,689	1,820	2,093	2,367	2,585	2,787	2,960	3,198	3,479	3,579	3,725
Embezzlement	118	146	139	166	308	414	536	758	920	1,051	1,350	1,393	1,360	1,477	1,587	1,638	1,687	1,849	1,979	1,943	2,018
False pretenses/Swindle/ Confidence game	210	325	303	303	501	639	831	1,169	1,462	1,647	2,028	2,196	2,599	2,829	3,081	3,214	3,312	3,510	3,765	3,722	3,800
Impersonation	68	124	162	140	160	221	339	486	624	808	1,094	1,335	1,610	1,868	2,136	2,217	2,408	2,627	2,895	2,991	3,118
Welfare fraud	16	14	180	136	42	44	47	91	101	111	112	124	140	178	209	209	209	253	223	266	302
Wire fraud	23	23	54	48	69	94	137	188	236	300	359	423	518	628	787	850	941	962	1,064	1,086	1,103

Source: Federal Bureau of Investigation, National Incident-Based Reporting System, 1991–2011.

Note: ¹ Assault consists of aggravated assault, simple assault, and intimidation.

Table A2
Number of Incidents by Year: 2000–2011

Offense	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
<i>Nonfraudulent</i>												
Assault ¹	553,646	608,813	603,403	612,431	628,526	641,673	634,700	646,256	633,788	624,222	614,890	583,261
Burglary/Breaking and entering	237,936	274,376	287,555	284,149	279,438	286,437	292,151	288,642	293,525	292,922	292,792	285,641
Larceny/Theft	912,495	1,008,864	1,015,680	1,006,780	998,934	990,742	958,284	951,848	952,301	924,803	901,198	873,683
Motor vehicle theft	113,030	132,102	134,031	133,233	129,053	130,599	122,387	113,331	102,157	86,795	80,129	75,967
Robbery	30,839	37,849	38,289	38,430	37,863	40,421	42,517	42,658	43,085	39,628	35,458	32,544
<i>Fraudulent</i>												
Counterfeiting/Forgery	41,335	56,298	56,891	58,047	63,416	64,194	55,820	49,755	45,139	42,572	38,885	35,103
Credit card/ATM fraud	14,288	20,348	21,208	21,863	24,895	28,657	32,601	36,739	39,684	39,662	40,869	41,024
Embezzlement	11,659	12,687	11,256	11,001	11,606	12,120	12,746	13,179	12,688	9,946	9,089	8,709
False pretenses/Swindle/Confidence game	30,945	43,391	45,785	50,282	56,151	60,971	60,349	60,641	58,003	55,444	53,072	52,126
Impersonation	5,650	8,882	10,242	12,194	14,879	17,393	19,404	20,903	23,608	22,692	21,161	21,638
Welfare fraud	83	66	92	75	51	62	55	96	125	103	111	113
Wire fraud	413	561	610	563	815	963	961	1,199	1,375	1,515	1,450	1,699

Source: Federal Bureau of Investigation, National Incident-Based Reporting System, 2000–2011.

Note: ¹ Assault consists of aggravated assault, simple assault, and intimidation.

Figure A1
Assault
MODWT Decomposition

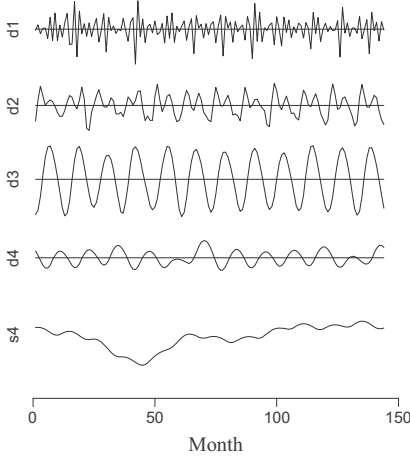


Figure A2
Assault
Variance Decomposition

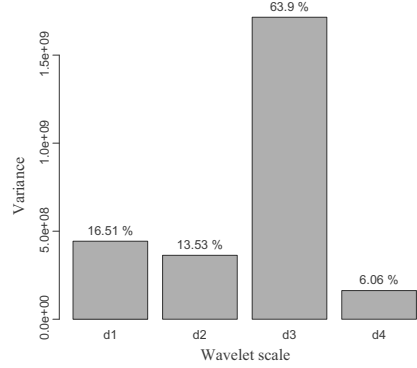


Figure A3
Burglary/Breaking and Entering
MODWT Decomposition

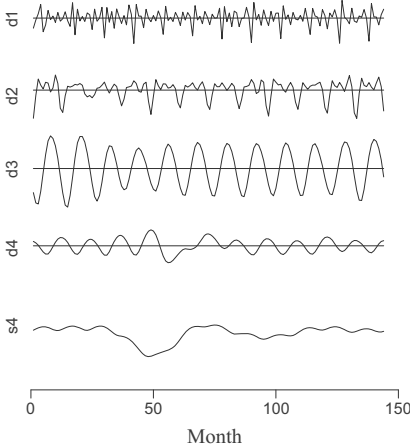


Figure A4
Burglary/Breaking and Entering
Variance Decomposition

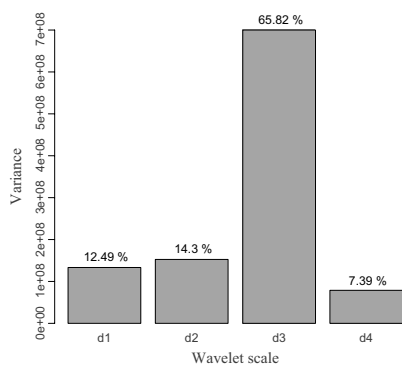


Figure A5
Larceny/Theft
MODWT Decomposition

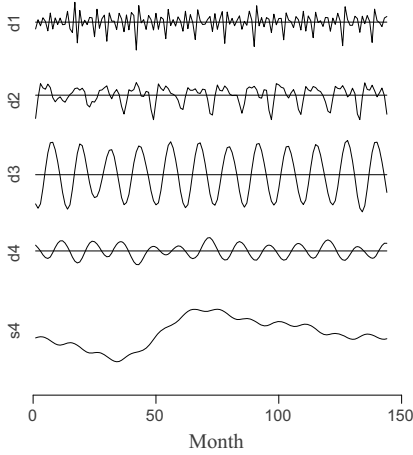


Figure A6
Larceny/Theft
Variance Decomposition

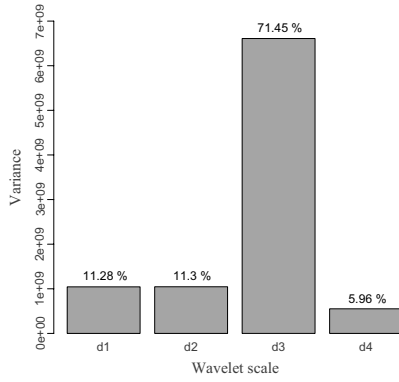


Figure A7
Motor Vehicle Theft
MODWT Decomposition

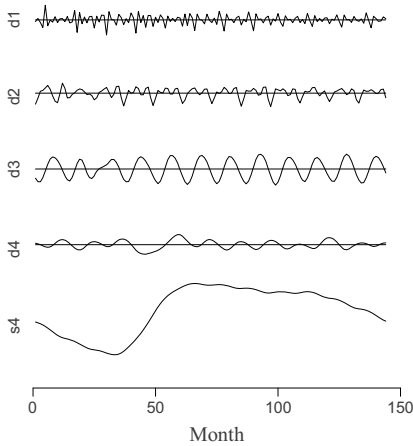


Figure A8
Motor Vehicle Theft
Variance Decomposition

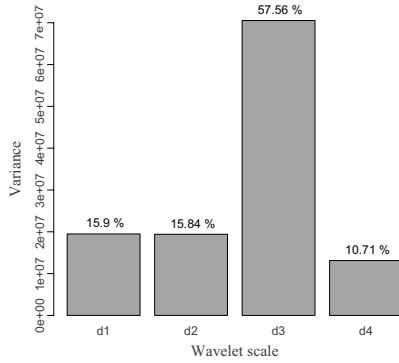


Figure A9
Robbery
MODWT Decomposition

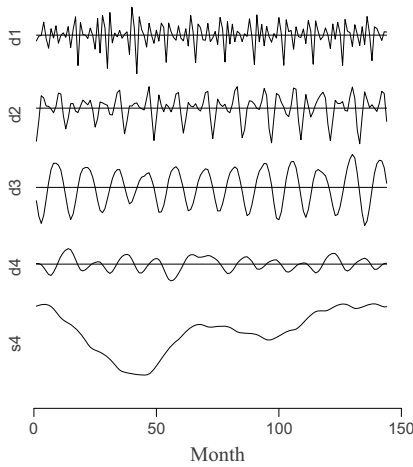


Figure A10
Robbery
Variance Decomposition

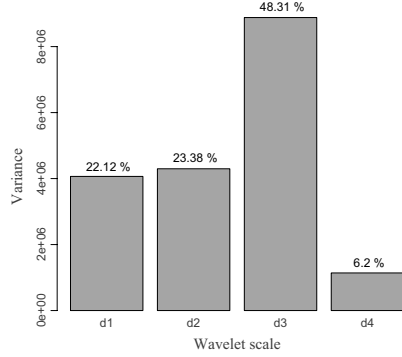


Figure A11
Counterfeiting/Forgery
MODWT Decomposition

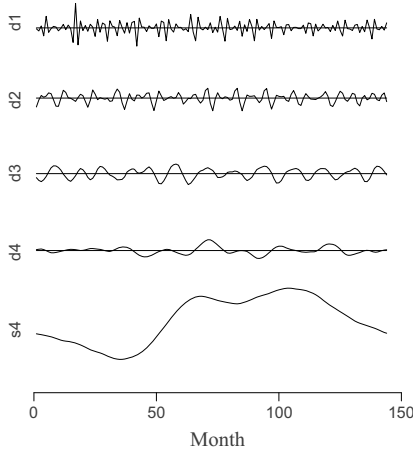


Figure A12
Counterfeiting/Forgery
Variance Decomposition

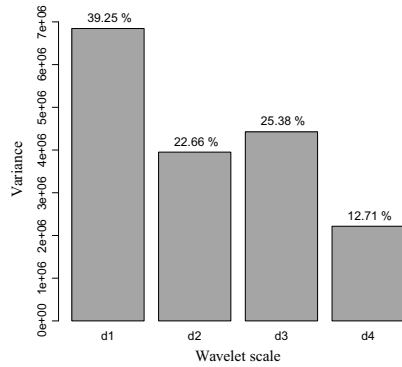


Figure A13
Credit Card/ATM Fraud
MODWT Decomposition

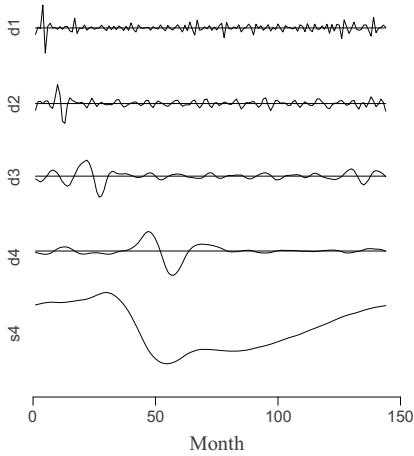


Figure A14
Credit Card/ATM Fraud
Variance Decomposition

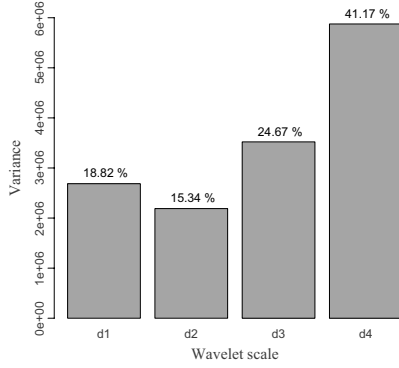


Figure A15
Embezzlement
MODWT Decomposition

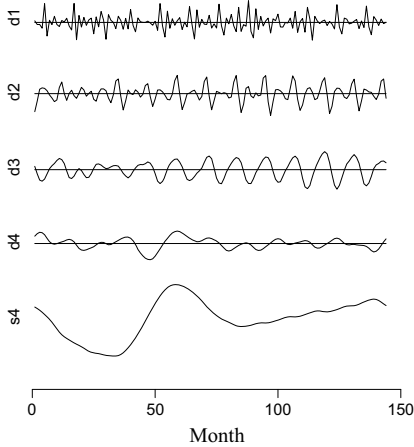


Figure A16
Embezzlement
Variance Decomposition

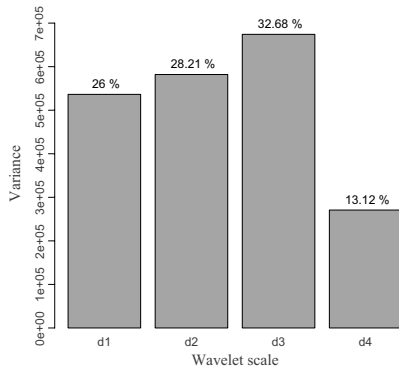


Figure A17
Impersonation
MODWT Decomposition

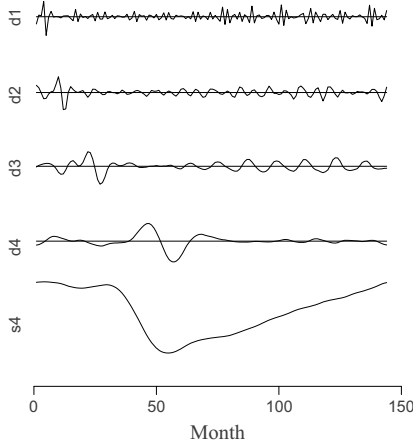


Figure A18
Impersonation
Variance Decomposition

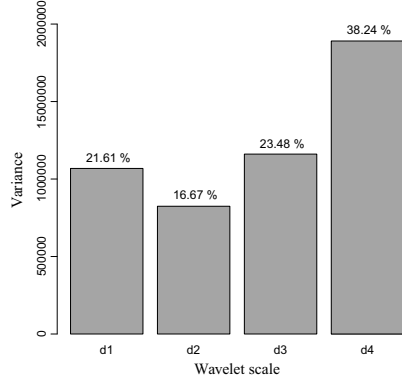


Figure A19
False Pretenses/Swindle/Confidence Game
MODWT Decomposition

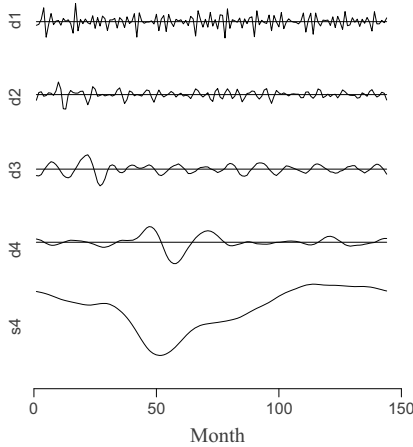


Figure A20
False Pretenses/Swindle/Confidence Game
Variance Decomposition

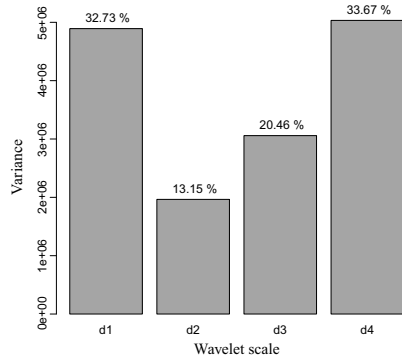


Figure A21
Welfare Fraud
MODWT Decomposition

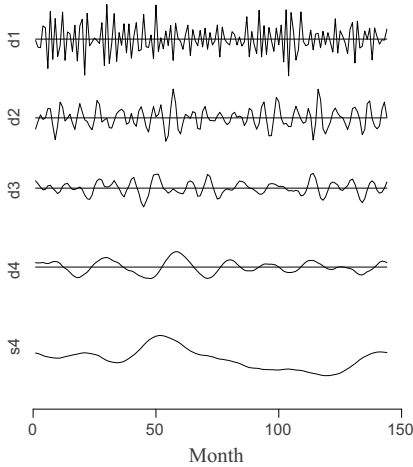


Figure A22
Welfare Fraud
Variance Decomposition

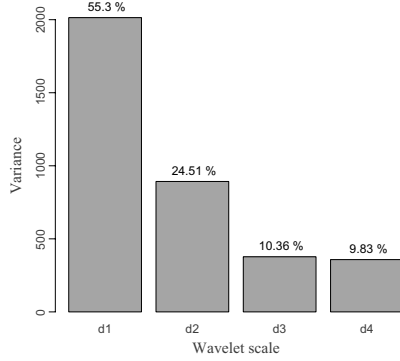


Figure A23
Wire Fraud
MODWT Decomposition

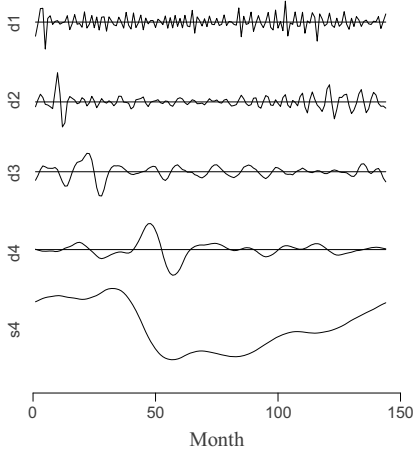


Figure A24
Wire Fraud
Variance Decomposition

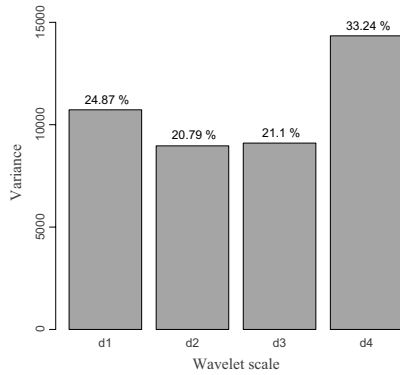


Figure A25
Wavelet Multiple Correlation
Nonfraudulent Offenses

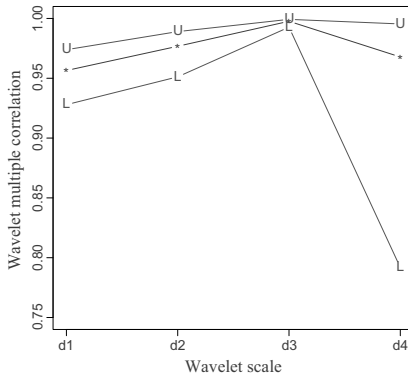
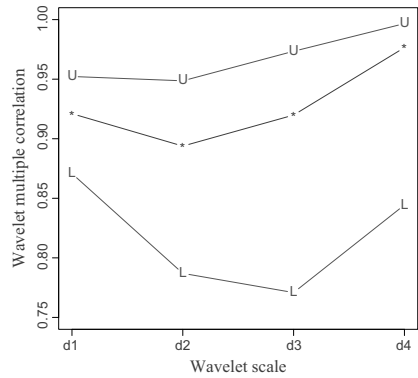


Figure A26
Wavelet Multiple Correlation
Fraudulent Offenses



Note: The upper and lower bounds indicate the 95-percent confidence interval.

Figure A27
Random Sequence of 144 Numbers
MODWT Decomposition

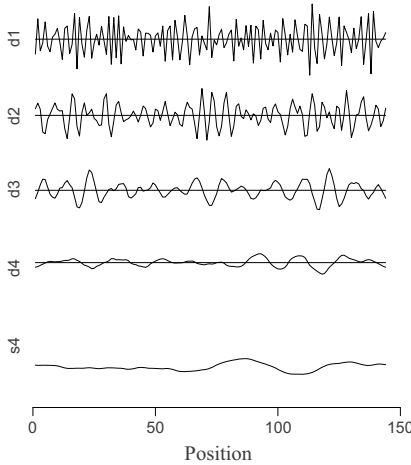
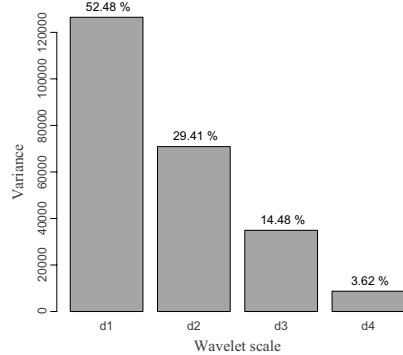


Figure A28
Random Sequence of 144 Numbers
Variance Decomposition



References

Aristotle (1984), "Constitution of Athens," transl. by F. G. Kenyon, in: J. Barnes (ed.), *The Complete Works of Aristotle: The Revised Oxford Translation*, Vol. 2, Princeton University Press, Princeton (NJ), pp. 2341–2383.

Association of Certified Fraud Examiners (2010), "Report to the Nations on Occupational Fraud and Abuse," Association of Certified Fraud Examiners, Austin (TX), available at http://www.acfe.com/uploadedfiles/acfe_website/content/documents/rtn-2010.pdf, last accessed February 2, 2016.

- Berentsen, A., E. Bruegger, and S. Loertscher (2008), "On Cheating, Doping and Whistle-blowing," *European Journal of Political Economy*, 24(2), 415–436.
- and Y. Lengwiler (2004), "Fraudulent Accounting and Other Doping Games," *Journal of Institutional and Theoretical Economics (JITE)*, 160(3), 402–415.
- Caplan, D. (1999), "Internal Controls and the Detection of Management Fraud," *Journal of Accounting Research*, 37(1), 101–137.
- Constantine, W., and D. Percival (2010), "wmts: Wavelet Methods for Time Series Analysis," Software Package, Version 2.0-0, to Percival and Walden (2000), available at <http://cran.r-project.org/web/packages/wmts/index.html>, last accessed February 2, 2016.
- Corona, C., and R. S. Randhawa (2010), "The Auditor's Slippery Slope: An Analysis of Reputational Incentives," *Management Science*, 56(6), 924–937.
- Crowley, P. M. (2007), "A Guide to Wavelets for Economists," *Journal of Economic Surveys*, 21(2), 207–267.
- Feldman, R. (2001), "An Economic Explanation for Fraud and Abuse in Public Medical Care Programs," *The Journal of Legal Studies*, 30(S2), 569–577.
- Fellingham, J. C., and D. P. Newman (1985), "Strategic Considerations in Auditing," *The Accounting Review*, 60(4), 634–650.
- Fernández-Macho, J. (2011), "Wavelet Multiple Correlation and Cross-Correlation: A Multiscale Analysis of Euro Zone Stock Markets," Working Paper BI-2095/2011, Universidad del País Vasco, Leioa, Bizkaia, available at <http://www.ehu.es/documents/30766/5a7da718-8abd-48c1-a684-edc2285c5fdb>.
- (2012), "wavemulcor: Wavelet Routine for Multiple Correlation," Software Package, Version 1.2, available at <https://cran.r-project.org/web/packages/wavemulcor/index.html>, last accessed February 2, 2016.
- Freeman, S., J. Grogger, and J. Sonstelie (1996), "The Spatial Concentration of Crime," *Journal of Urban Economics*, 40(2), 216–231.
- Friedman, D. (1991), "Evolutionary Games in Economics," *Econometrica*, 59(3), 637–666.
- Frontain, M. (2010), "Enron Corporation," in: *Handbook of Texas Online*, Texas State Historical Association, Austin (TX), available at <https://www.tshaonline.org/handbook/online/articles/doi08>, last accessed February 1, 2016.
- Hansen, S. C. (1993), "Strategic Sampling, Physical Units Sampling, and Dollar Units Sampling," *The Accounting Review*, 68(2), 323–345.
- Henriques, D. B. (2010), "Deal Recovers \$7.2 Billion for Madoff Fraud Victims," *The New York Times*, December 17, available at http://dealbook.nytimes.com/2010/12/17/big-settlement-with-madoff-investor-near/?_r=0, accessed on February 1, 2016.
- Hopkins, E. (2002), "Two Competing Models of How People Learn in Games," *Econometrica*, 70(6), 2141–2166.
- Hyman, D. A. (2001), "Health Care Fraud and Abuse: Market Change, Social Norms, and the Trust 'Reposed in the Workmen'," *The Journal of Legal Studies*, 30(S2), 531–567.
- Krüger, N. A. (2011), "The Impact of Economic Fluctuations on Crime: A Multiscale Analysis," *Applied Economics Letters*, 18(2), 179–182.
- Lucas, Jr., R. E. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4(2), 103–124.
- Luenberger, D. G. (1979), *Introduction to Dynamic Systems: Theory, Models, & Applications*, John Wiley & Sons, New York.
- Lui, F. T. (1986), "A Dynamic Model of Corruption Deterrence," *Journal of Public Economics*, 31(2), 215–236.
- Matsumura, E. M., and R. R. Tucker (1992), "Fraud Detection: A Theoretical Foundation," *The Accounting Review*, 67(4), 753–782.
- McDowall, D., C. Loftin, and M. Pate (2012), "Seasonal Cycles in Crime, and their Variability," *Journal of Quantitative Criminology*, 28(3), 389–410.
- Montgomery, J. D. (2010), "Intergenerational Cultural Transmission as an Evolutionary Game," *American Economic Journal: Microeconomics*, 2(4), 115–136.

- Noe, T. H., and M. J. Rebelló (1994), "The Dynamics of Business Ethics and Economic Activity," *The American Economic Review*, 84(3), 531–547.
- Percival, D. T., and A. T. Walden (2000), *Wavelet Methods for Time Series Analysis*, Cambridge University Press, Cambridge.
- Povel, P., R. Singh, and A. Winton (2007), "Booms, Busts, and Fraud," *The Review of Financial Studies*, 20(4), 1219–1254.
- Raphael, S., and R. Winter-Ebmer (2001), "Identifying the Effect of Unemployment on Crime," *The Journal of Law & Economics*, 44(1), 259–283.
- Reinstein, A., and M. E. Bayou (1998), "A Comprehensive Structure to Help Analyze, Detect, and Prevent Fraud," available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=172055.
- Samuelson, L. (1998), *Evolutionary Games and Equilibrium Selection*, The MIT Press, Cambridge (MA).
- Shonkwiler, J. S., and T. H. Spreen (1986), "Statistical Significance and Stability of the Hog Cycle," *Southern Journal of Agricultural Economics*, 18(2), 227–233.
- Simić, D. (2005), "Reducing Fraud in Electronic Payment Systems," The 7th Balkan Conference on Operational Research, available at <http://fmi.unibuc.ro/balkan-conf/CD/Section5/SimicDejan.pdf>.
- Singleton, T. W., and A. J. Singleton (2010), *Fraud Auditing and Forensic Accounting*, 4th edn., John Wiley & Sons, Hoboken (NJ).
- Sutter, D. (2003), "Detecting and Correcting Election Fraud," *Eastern Economic Journal*, 29(3), 433–451.
- Taylor, P. D., and L. B. Jonker (1978), "Evolutionary Stable Strategies and Game Dynamics," *Mathematical Biosciences*, 40(1–2), 145–156.
- U.S. Department of Justice, Federal Bureau of Investigation, and Criminal Justice Information Services Division (2000), "Uniform Crime Reporting: National Incident-Based Reporting System, Volume 1: Data Collection Guidelines," available at <https://www2.fbi.gov/ucr/nibrs/manuals/v1all.pdf>, last accessed February 2, 2016.

Jiong Gong
University of International
Business and Economics
East Huixin Street 10
100029 Beijing
P.R. China
johngong@gmail.com

R. Preston McAfee
Microsoft
Bldg. 34
One Microsoft Way
Redmond, WA 98052
U.S.A.
preston@mcafee.cc

Michael A. Williams
Competition Economics LLC
Suite 510
2000 Powell Street
Emeryville, CA 94608
U.S.A.
mwilliams@c-econ.com