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EXTRACTING THE SURPLUS IN THE COMMON-VALUE AUCTION

BY R. PRESTON MCAFEE, JOHN MCMILLAN, AND PHILIP J. RENY[1]

1. SELLING AN ITEM WITH A COMMON VALUE

CONSIDER THE SALE of a unique item with a well-defined true value which no one knows (like mineral rights). What selling procedure should the owner choose?[2] The mechanism we devise takes advantage of the fact that a common-value sale has an efficient outcome regardless of which of the potential buyers receives the item, since all potential buyers value the item identically ex post. Thus any mechanism that extracts all the rents must be optimal for the seller.[3]

We consider the following simple mechanism. After each potential buyer has received his private signal about the true value of the item and committed himself to participate in the sale, the seller arbitrarily selects two of them. The seller then asks one, j, to report his signal, and offers the item to the other, i, for a price $z(s_j)$, where s_j is j's report. Since the payoffs to all agents are independent of their own actions, this mechanism is (weakly) incentive compatible. The mechanism satisfies the individual rationality constraints, i.e., the potential buyers are willing to participate, if the price function z is such that they expect nonnegative rents. In addition, if these rents are zero, the mechanism is optimal from the seller's point of view.

Does such a price function exist? We shall prove that it is always possible for the seller to extract arbitrarily close to the full expected rents. Moreover, shall we give two sufficient conditions for exactly all of the expected rents to be extractable. We also give an example in which the seller can extract almost all, but not all, of the rents. Simple contracts work: the seller can without loss choose the price function to be a piecewise linear function or a step function.

To motivate the results, consider a special case. Suppose the bidders can construct unbiased estimates $z(s_i)$ of v from their signals s_i and that s_i is independent of s_j, given v. Then[4]

$$v = \int z(s_j) f(s_j | v)\, ds_j. \tag{1}$$

The price function $z(s_j)$ extracts all the rents, since

$$E\left[z(s_j) | s_i\right] = E[v | s_i]. \tag{2}$$

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[2] Most of the literature on the design of optimal selling mechanisms assumes independent private values; i.e., valuations are drawn independently from some distribution $H(v)$ (Harris and Raviv (1981), Maskin and Riley (1984), Matthews (1983), McAfee and McMillan (1987b), Milgrom (1985), Riley and Samuelson (1981)). The model of Myerson (1981) is more general than independent private values in allowing a limited form of correlation among valuations. Crémer and McLean (1985) showed that with correlated private values (bidder i's value is of the form $v_i(s)$, where s is the vector of all bidders' signals) and a finite value space, the seller can extract all of the rents by using a combination of a lottery plus a Vickrey auction. For a detailed review of this literature, see McAfee and McMillan (1987a).

[3] It follows that the mechanism is sensitive to the common-value assumption and will not work for the more general case of affiliated values (Milgrom and Weber (1982)).

[4] Equation (1) is a Fredholm equation of the first kind. General existence of solutions to such equations is examined in the mechanism-design context by McAfee and Reny (1988); a particular case is examined by Melamud and Reichelstein (1986) and Caillaud, Guesnerie, and Rey (1988).

We shall show that in one sense this example is typical, in that the common-value environment facilitates rent extraction; but in another sense it oversimplifies the situation, in that in general the seller can extract almost all, but not necessarily all, of the surplus.

This mechanism suffers from a multiple-equilibrium problem. While agent j cannot do better than report his value correctly, he is no worse off if he lies: truth-telling is only a weakly optimal strategy. This can, however, be corrected at arbitrarily small cost to the principal: agent j can be given a strict incentive to tell the truth. Let $\delta > 0$ be small and consider a revelation game R in which each bidder has a strict incentive to tell the truth (e.g. the revelation equivalent of a second-price auction). The seller, having received the reports, uses them in the revelation game R with probability δ, and in the mechanism described above with probability $1 - \delta$. This provides a strict incentive for truth-telling, at a cost to the seller that vanishes as $\delta \to 0$.

2. RENT EXTRACTION

With the unknown true value of the item denoted by v, suppose that each potential buyer has received (without cost) a signal s_i independently generated from the distribution $F(s_i|v)$. The corresponding density, $f(s_i|v)$, is assumed to exist and to be strictly positive and continuous on $[0,1] \times [0,1]$.[5] Let the uncertainty about v be given by a measure G with support that is a subset of $[0,1]$. The seller and all potential buyers are assumed to be risk neutral.

We assume that, after having observed his signal, been told the selling mechanism, and agreed to participate, the buyer, i, is committed to pay $z(s_j)$ whatever the realization of the other's report s_j: thus he cannot back out if the price appears to him to be atypically high. (This is stricter than take-it-or-leave-it pricing; the buyer must take it.) Since, however, the potential buyers have the option of not participating, the mechanism must offer them rents that are nonnegative conditional on their own signals. The chosen buyer's expected rents, given his signal s_i, are

$$\pi(s_i) = \int_0^1 \left[v - \int_0^1 z(s) f(s|v)\, ds \right] \frac{f(s_i|v)}{\int_0^1 f(s_i|u)\, dG(u)}\, dG(v). \tag{3}$$

Remarkably, the problem of extracting the rents, that is,

$$(\forall\, s_i) \qquad \pi(s_i) = 0, \tag{4}$$

can be transformed into a minimum-norm problem. Consider the $L^2([0,1], G)$ dot product and norm

$$(x, y) = \int_0^1 x(v) y(v)\, dG(v); \quad \|x\| = (x, x)^{1/2} \tag{5}$$

and the set

$$X = \left\{ x \in L^2([0,1], G) | \exists z \in L^2([0,1], \lambda), \quad x(v) = \int_0^1 z(s) f(s|v)\, ds \right\} \tag{6}$$

where λ is Lebesgue measure, and $L^2([0,1], \lambda)$ will be denoted $L^2(\lambda)$, and similarly $L^2(G)$ for $L^2([0,1], G)$.

[5] If the support of $f(\cdot\,|v)$ is monotonic in v, the rents can be fully extracted, since the problem of rent extraction reduces to the solution of a Volterra equation. See Hochstadt (1973).

LEMMA: *$\exists z \in L^2(\lambda)$ satisfying $(\forall s_i)\pi(s_i) = 0$ if and only if*

$$(7) \qquad \min_{x \in X} \|x - v\|$$

has a solution.[6]

PROOF: x is the solution to (7) if and only if (by Theorem 1.4.1 of Balakrishnan (1981, p. 9), noting X is a linear subspace)

$$(\forall \hat{x} \in X) \quad (x - v, \hat{x}) = 0 \qquad \text{iff} \quad (\text{by } (5))$$

$$(\forall \hat{x} \in X) \quad \int_0^1 (v - x(v))\hat{x}(v)\, dG(v) = 0 \qquad \text{iff} \quad (\text{since } \hat{x} \in X, (6))$$

$$\forall \hat{z} \in L^2(\lambda) \quad \int_0^1 (v - x(v)) \int_0^1 \hat{z}(s) f(s|v)\, ds\, dG(v) = 0 \qquad \text{iff}$$

(by Fubini's Theorem, applicable since $(v - x(v))\hat{z}(s)f(s|v) \in L^1([0,1]^2, \lambda \times G)$)

$$\forall \hat{z} \in L^2(\lambda) \quad \int_0^1 \hat{z}(s) \int_0^1 (v - x(v)) f(s|v)\, dG(v)\, ds = 0 \qquad \text{iff}$$

(by the Fundamental Lemma of the Calculus of Variations)

$$\text{a.e. } s_i \in [0,1], \quad \int_0^1 [v - x(v)] f(s_i|v)\, dG(v) = 0 \qquad \text{iff} \quad (\text{by } (6) \text{ and } x \in X)$$

$$\text{a.e. } s_i \in [01], \quad \int_0^1 \left[v - \int_0^1 z(s) f(s|v)\, ds \right] f(s_i|v)\, dG(v) = 0$$

$$\text{iff} \quad (\text{by continuity of } f)$$

$$\forall s_i \in [0,1], \quad \int_0^1 \left[v - \int_0^1 z(s) f(s|v)\, ds \right] f(s_i|v)\, dG(v) = 0 \qquad \text{iff}$$

$$\forall s_i \in [0,1], \qquad \pi(s_i) = 0. \qquad Q.E.D.$$

The main result of the paper is that the seller can always extract almost all of the rents.

THEOREM: *$\forall\ \varepsilon > 0$, $\exists z \in L^2(\lambda)$ such that $\pi(s) \in [0, \varepsilon]$ $\forall s \in [0,1]$, where $\pi(s)$ denotes a buyer's expected rent as defined by* (3).

The proof has two steps. In the first, a sequence $\phi_n \in X$ is constructed, with ϕ_n converging to a solution (not necessarily in X) of (7), with X replaced with the closure of X. There is an associated sequence of profit functions

$$(8) \qquad \Delta_n(u) = \int_0^1 (v - \phi_n(v)) f(u|v)\, dG(v),$$

which we prove to be equicontinuous and to converge uniformly to zero. In the second step, ϕ_n and Δ_n are used to construct a sequence $\hat{z}_n$ of price functions, with associated

[6] v will be used to denote both the identity function, as in (7), and a dummy of integration, as in (5).

profit functions π_n, which satisfy individual rationality and also converge to zero uniformly.

PROOF: Let $\overline{X}$ denote the closure of X in $L^2(G)$. Clearly, $\overline{X}$ is a closed linear subspace of $L^2(G)$. Hence, by Theorem 1.4.1 of Balakrishnan (1981, p. 9), (7) has a solution with X replaced by $\overline{X}$. Denote this solution by ϕ. Hence, for every $x \in \overline{X}$, we have $(\phi - v, x) = 0$, i.e.,

$$(9) \qquad \int_0^1 x(v)(\phi(v) - v)\, dG = 0 \quad \forall\, x \in \overline{X}.$$

Now, since $\phi \in \overline{X}$, $\exists \phi_n \in X$ such that $\phi_n \overset{L^2}{\rightarrow} \phi$ ($\overset{L^2}{\rightarrow}$ denotes convergence in $L^2(G)$). By the continuity of the inner product, we have $\Delta_n(u) \rightarrow \int_0^1 (v - \phi(v)) f(u|v)\, dG$ for all u, where Δ_n is given in (8). Let $\Delta(u) \equiv \int_0^1 (v - \phi(v)) f(u|v)\, dG$ so that $\Delta_n(u) \rightarrow \Delta(u)$ for all $u \in [0,1]$.

Putting $x(v) = \int_0^1 z(s) f(s|v)\, ds$ in (9) above, we have (for any $z \in L^2(\lambda)$)

$$(10) \qquad \int_0^1 \left[\int_0^1 z(s) f(s|v)\, ds\right](v - \phi(v))\, dG = 0,$$

or, using Fubini's Theorem (applicable since $z(s) f(s|v)(v - \phi(v)) \in L^1([0,1]^2, \lambda \times G)$),

$$(11) \qquad \int_0^1 z(s)\left[\int_0^1 (v - \phi(v)) f(s|v)\, dG\right] ds = 0,$$

i.e.,

$$(12) \qquad \forall\, z \in L^2(\lambda), \qquad \int_0^1 z(s)\Delta(s)\, ds = 0.$$

Hence, $\Delta(s) = 0$ a.e. In fact $\Delta(s) = 0$ for all $s \in [0,1]$ since by definition $\Delta(\cdot)$ is continuous (by the continuity of $f(\cdot\,|\,\cdot)$). Hence, for all $u \in [0,1]$, we have

$$(13) \qquad \Delta_n(u) = \int_0^1 (v - \phi_n(v)) f(u|v)\, dG \rightarrow \Delta(u) = 0.$$

Since f is continuous and $\{v - \phi_n\}$ is bounded in $L^2(G)$, it is easy to show that $\{\Delta_n\}$ is uniformly bounded and equicontinuous (see Hochstadt (1973, p. 51)). In particular, letting $\delta_n \equiv \min_{u \in [0,1]} \Delta_n(u)$, and recalling that $\Delta_n(u) \rightarrow 0$ for all u, we have that $\delta_n \rightarrow 0$. Note that by construction $\Delta_n(u) + |\delta_n| \geqslant 0$ for all $u \in [0,1]$ and all n. Let

$$(14) \qquad \alpha_n \equiv \frac{|\delta_n|}{\min\limits_u \int_0^1 f(u|v)\, dG}.$$

(This is well defined since $\int_0^1 f(\cdot\,|v)\, dG$ is continuous on $[0,1]$ and has a minimum which is bounded away from zero since $\int_0^1 f(u|v)\, dG > 0$, $\forall u \in [0,1]$.) Clearly, $\alpha_n \rightarrow 0$. Define $\{\hat{z}_n\} \subseteq L^2(\lambda)$ as follows: Since $\{\phi_n\} \subseteq X$, $\exists \{z_n\} \subseteq L^2(\lambda)$ such that $\phi_n(v) = \int z_n(s) f(s|v)\, ds$ for all n. Put $\hat{z}_n(s) \equiv z_n(s) - \alpha_n$ for all $s \in [0,1]$, and all n.

We now show that $\hat{z}_n$ is individually rational for every n. Let π_n denote the profits associated with $\hat{z}_n$, as given in (3).

$$\text{(15)} \qquad \pi_n(u)\cdot C_1 \geqslant \int_0^1\left(v-\int_0^1 \hat{z}_n(s)f(s|v)\,ds\right)f(u|v)\,dG,$$

$$\text{where} \quad C_1 \equiv \max_{u\in[0,1]}\int_0^1 f(u|v)\,dG>0,$$

$$=\int_0^1\left(v-\int_0^1(z_n(s)-\alpha_n)f(s|v)\,ds\right)f(u|v)\,dG$$

$$=\int_0^1\left(v-\int_0^1 z_n(s)f(s|v)\,ds\right)f(u|v)\,dG+\alpha_n\int_0^1 f(u|v)\,dG$$

$$=\int_0^1(v-\phi_n(v))f(u|v)\,dG+|\delta_n|\frac{\int_0^1 f(u|v)\,dG}{\min_{u\in[0,1]}\int_0^1 f(u|v)\,dG}$$

$$\geqslant \Delta_n(u)+|\delta_n|\geqslant 0.$$

Hence $\hat{z}_n$ is individually rational for every n and every $u\in[0,1]$.

Finally we must show that for large enough n, $\hat{z}_n$ extracts an arbitrarily large fraction of the surplus. First, observe that $\int_0^1\hat{z}_n(s)f(s|\cdot)\,ds \xrightarrow{L^2} \phi$ since $\int_0^1\hat{z}_n(s)f(s|v)\,ds=\phi_n(v)-\alpha_n$ $\forall n, v$. Denoting $\int_0^1\hat{z}_n(s)f(s|v)\,ds$ by $\hat{\phi}_n(v)$ we then have $\hat{\phi}_n \xrightarrow{L^2} \phi$ so that

$$\text{(16)} \qquad \pi_n(u)=\int_0^1(v-\hat{\phi}_n(v))h(u,v)\,dG, \qquad \text{where} \quad h(u,v)\equiv\frac{f(u|v)}{\int_0^1 f(u|v)\,dG}.$$

$$\pi_n(u)\to\int_0^1(v-\phi(v))h(u,v)\,dG \qquad \text{(by continuity of the inner product)}$$

$$=0 \quad \text{for every } u\in[0,1] \ (\text{since } \Delta(u)=0 \text{ for every } u\in[0,1]).$$

Moreover, since $\{\hat{\phi}_n\}$ is clearly bounded and $h(\cdot,\cdot)$ is continuous, we have $\{\pi_n\}$ is a uniformly bounded and equicontinuous family. Hence $\pi_n(u)\to 0$ uniformly, i.e., $(\forall\varepsilon>0)$ $(\exists N)$ such that $\forall n\geqslant N$, $\forall u\in[0,1]$, $|\pi_n(u)|<\varepsilon$. Since, from (14), $\pi_n(u)\geqslant 0$, we have $0\leqslant\pi_n(u)<\varepsilon$, as desired. *Q.E.D.*

By Theorem 1.4.1 of Balakrishnan (1981, p. 9), a solution to (7) exists if X is closed. Note that if the minimized value of the norm in (7) is zero then there is a trivial rent-extraction problem, as described in the introduction (equation (1)). The more remarkable result is that even if the minimum norm is not zero (i.e. there does not exist an unbiased estimate $z(s_j)$ of v) but merely a solution to (7) exists, then there is still a mechanism that extracts exactly all of the rents. In Examples 1 and 2, we give two sufficient conditions for X to be closed, so that $\bar{X}=X$, and hence for the rents to be fully extractable.

Example 1: If

$$\text{(17)} \qquad f(s|v)=\sum_{i=1}^{n}a_i(s)b_i(v),$$

then $\exists z\in L^2(\lambda)$ satisfying (4).

PROOF: Viewed as an integral operator on $L^2(G)$, $f(s|v)=\sum_{i=1}^{n}a_i(s)b_i(v)$ is degenerate, and hence by an obvious modification of Theorem 12 of Hochstadt (1973, p. 60) (modifying from $L^2(\lambda)$ to $L^2(G)$), X is finite dimensional. Since X is hence a finite-dimensional linear subspace of a normed linear space (namely $L^2(G)$), Theorem 4.3.2 of Friedman (1970, p. 132) shows that X is closed. *Q.E.D*

EXAMPLE 2: If G has finite support, then $\exists z \in L^2(\lambda)$ satisfying (4).

PROOF: Let

$$X \equiv \left\{ x \in R^m \mid x_i = \int_0^1 z(s) f(s|v_i)\, ds \text{ for some } z \in L^2(\lambda) \right\}, \tag{18}$$

where

$$\begin{aligned} &\operatorname{supp} G = \{v_1, v_2, \dots, v_m\}, \\ &G(v_i) = \sum_{k=1}^{i} \alpha_k, \quad \text{and} \\ &\alpha_k \geqslant 0 \ \forall k \quad \text{and} \quad \sum_{k=1}^{m} \alpha_k = 1. \end{aligned} \tag{19}$$

In this case, the norm defined in (5) reduces to $\|x\| \equiv (\sum_{i=1}^{m}\alpha_i x_i^2)^{1/2}$, $\forall x \in R^m$. Since X is clearly a finite-dimensional linear subspace of R^m, $(X, \|\cdot\|)$ is a finite-dimensional normed linear subspace and X is therefore closed. *Q.E.D.*

Finally, we give an example showing that, under the hypotheses of the theorem, no stronger result exists.There are cases in which full rent extraction is impossible; the best the seller can do, using this type of mechanism, is to extract almost all the rents.

EXAMPLE 3: Suppose that

$$f(s|v) = \frac{1}{2\pi} + \frac{1}{\pi^3}\sum_{k=1}^{\infty}\left(\frac{\sin ks \sin kv + \cos ks \cos kv}{k^2}\right),$$

so that f: $[-\pi, \pi] \times [-\pi, \pi] \to R_+$ is continuous and strictly positive and

$$\int_{-\pi}^{\pi} f(s|v)\, ds = 1 \quad \text{for all} \quad v \in [-\pi, \pi].$$

Further suppose that

$$g(v) = \begin{cases} \dfrac{1}{2\pi}, & v \in [-\pi, \pi], \\ 0, & \text{otherwise.} \end{cases}$$

Then full rent extraction is impossible using the mechanism of the theorem.

PROOF: Since

(a) $$\int_{-\pi}^{\pi} 1 \cdot f(s|v)\, ds = 1,$$

(b) $$\int_{-\pi}^{\pi} \pi^2 k^2 \sin ks f(s|v)\, ds = \sin kv,$$

(c) $$\int_{-\pi}^{\pi} \pi^2 k^2 \cos ks f(s|v)\, ds = \cos kv,$$

we have:

$$\{1, \sin x, \cos x, \sin 2x, \cos 2x, \dots\} \subseteq X,$$

where $L^2(\lambda)$ is now used to denote $L^2([-\pi, \pi], \lambda)$, and

$$X = \left\{ x \in L^2(\lambda) | x(v) = \int_{-\pi}^{\pi} z(s) f(s|v)\, ds \text{ for some } z \in L^2(\lambda) \right\}.$$

Hence, $\overline{X} = L^2(\lambda)$, so that $v \in \overline{X}$; i.e. $\operatorname{argmin}_{x \in \overline{X}} \|x - v\| = v$. On the other hand, $v \notin X$ (shown below) so that $\min_{x \in X} \|x - v\|$ has no solution (in X). Since our rent extraction problem is equivalent to this minimum-norm problem, this implies that the seller cannot extract all of the rents.

It remains to show that $v \notin X$. For this, note that, viewed as an integral operator, $f(\cdot | \cdot)$ is compact and self-adjoint. Hence if $\{\phi_i\}_{i=0}^{\infty}$ denotes an orthonormal set of eigenfunctions of $f(\cdot | \cdot)$ and $\{\mu_i\}_{i=0}^{\infty}$ denotes the set of corresponding eigenvalues with $|\mu_0| \geqslant |\mu_1| \geqslant |\mu_2| \geqslant \dots$, then by Picard's Theorem (Hochstadt (1973, p. 108)), $v \in X$ implies

$$\sum_{i=0}^{\infty} \frac{|(v, \phi_i)|^2}{\mu_i^2} < \infty.$$

Now, corresponding to $f(\cdot | \cdot)$ we have

$$\{\phi_i\}_{i=0}^{\infty} = \left\{ \frac{1}{\sqrt{2\pi}}, \frac{\sin v}{\sqrt{\pi}}, \frac{\cos v}{\sqrt{\pi}}, \frac{\sin 2v}{\sqrt{\pi}}, \frac{\cos 2v}{\sqrt{\pi}}, \dots \right\},$$

and $\mu_0 = 1$,

$$\mu_i = \begin{cases} \dfrac{1}{\pi^2\sqrt{\pi}} \cdot \dfrac{4}{(i+1)^2} & (i = 1, 3, 5, \dots), \\ \dfrac{1}{\pi^2\sqrt{\pi}} \cdot \dfrac{4}{i^2} & (i = 2, 4, 6, \dots), \end{cases}$$

so that

$$\sum_{i=0}^{\infty} \frac{|(v, \phi_i)|^2}{\mu_i^2} = \sum_{i \text{ odd}} \frac{|(v, \phi_i)|^2}{\mu_i^2} = \sum_{k=1}^{\infty} \frac{\left(\dfrac{(-1)^{k+1} 2\sqrt{\pi}}{k} \right)^2}{\left(\dfrac{1}{\pi^2 \sqrt{\pi} k^2} \right)^2},$$

where the first equality appears since $\int_{-\pi}^{\pi} v\phi_i(v)\,dv = 0$ for all $i = 0, 2, 4, \ldots$, and the second equality is justified since

$$\int_{-\pi}^{\pi} v\phi_i(v)\,dv = \frac{(-1)^{(i+1)/2+1} 2\sqrt{\pi}}{\left(\frac{i+1}{2}\right)} \quad \text{for all} \quad i = 1, 3, 5, \ldots$$

(the change of variable $k = (i+1)/2$ was also employed). Hence,

$$\sum_{i=0}^{\infty} \frac{|(v, \phi_i)|^2}{\mu_i^2} = 4\pi^6 \sum_{k=0}^{\infty} k^2,$$

which clearly diverges. We conclude that $v \notin X$, as required. *Q.E.D.*

3. CONCLUSION AND EXTENSIONS

We have shown that, for the sale of an item with an unknown common value, the seller can always extract almost all of the rents. In particular, if an optimal auction exists it extracts all of the buyer's surplus. The seller extracts exactly all of the rents if either there is a finite number of possible values of the item, or the density function of signals conditional on true value satisfies a separability condition. We also illustrated, however, that with the mechanism used here full rent extraction is not always possible.

We note that the results continue to hold if a vector of characteristics $(c_1, \cdots, c_m)$ determines the common value v (i.e., $v(c_1, \cdots, c_m)$), where G now denotes the joint measure over $c = (c_1, \cdots, c_m)$ with support $[0,1]^m$. In addition, each bidder may receive a vector of signals $s = (s_1, \cdots, s_m)$. The mechanism works as before, except now when called upon a potential buyer reports his entire vector of signals. In this case, a potential buyer's rent as a function of signals is:

$$\text{(20)} \qquad \pi(s) = \int_{[0,1]^m} \left[v(c) - \int_{[0,1]^m} z(u) f(u|c)\,du \right] f(s|c)\,dG(c) \Bigg/ \int_{[0,1]^m} f(s|u)\,dG(u).$$

The functional form in the statement of Example 1 now becomes $\Sigma_{i=1}^n a_i(s) b_i(c)$.

If in all the above we replace X by

$$X' \equiv \left\{ x \in L^2(G) | x(v) = \int_0^1 z(s) f(s|v)\,ds \text{ for some } z \in Z \right\},$$

where Z is either the set of piecewise linear functions on $[0,1]$, or the set of step functions on $[0,1]$, the proofs go through verbatim. Hence in all cases the price function that the seller announces in advance can be a relatively simple function.

Three further extensions of the analysis are possible. First, if the seller himself draws a signal from the same distribution as the potential buyers, he need not use the report of the second potential buyer. Instead, he could make the price a function of his own signal, and proceed exactly as above, provided he can credibly commit not to misrepresent his signal. Second, if the seller has m units of the good and each potential buyer wants one unit, the seller optimally chooses m buyers at random and charges each of them $z(s_j)$. Third,[7] the n bidders' conditional densities need not be identical. One generalization requires only that there exist two bidders, say i and j, with conditional densities satisfying $f_i(s|v) = a(s) f_j(b(s)|v)$, where $a(s)$ is bounded away from zero and $b'(s) > 0$ on $[0,1]$, and b takes $[0,1]$ onto $[0,1]$.

[7] We thank Matt Spiegel for this observation.

The mechanism examined here superficially resembles the Vickrey auction, as the price paid by the buyer depends upon another potential buyer's report. However, the Vickrey auction is not optimal in the common-value case. (Milgrom and Weber (1982) show it is dominated by the English auction.) The essential difference between the mechanism analyzed here and the Vickrey auction is that, in the former, the two bidders are chosen arbitrarily. In contrast, in the latter, the bidders making the highest and second-highest reports are chosen; and the bidders' knowledge of this prevents the full extraction of the surplus.[8]

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REFERENCES

BALAKRISHAN, A. V. (1981): *Applied Functional Analysis*, 2nd Edition. New York: Springer-Verlag.

CAILLAUD, B., R. GUESNERIE, AND P. REY (1988): "Noisy Observation in Adverse Selection Models," Document du Travail No. 8802, Institut National de la Statistique et des Etudes Economiques.

CREMER, JACQUES, AND RICHARD P. MCLEAN (1985): "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist When Demands Are Interdependent," *Econometrica*, 53, 345–361.

FRIEDMAN, AVNER (1970): *Foundations of Modern Analysis*. New York: Dover.

HARRIS, MILTON, AND ARTUR RAVIV (1981): "Allocation Mechanisms and the Design of Auctions," *Econometrica*, 49, 1477–1499.

HOCHSTADT, HARRY (1973): *Integral Equations*. New York: Wiley.

KENNEY, ROY W., AND BENJAMIN KLEIN (1983): "The Economics of Block Booking," *Journal of Law and Economics*, 26, 49–54.

MASKIN, ERIC, AND JOHN G. RILEY (1983): "Optimal Auctions with Risk Averse Buyers," *Econometrica*, 52, 1473–1519.

MATTHEWS, STEVEN A. (1983): "Selling to Risk Averse Buyers with Unobservable Tastes," *Journal of Economic Theory*, 30, 370–400.

MCAFEE, R. PRESTON, AND JOHN MCMILLAN (1987a): "Auctions and Bidding," *Journal of Economic Literature*, 25, 699–738.

——— (1987b): "Auctions with a Stochastic Number of Bidders," *Journal of Economic Theory*, 43, 1–19.

MCAFEE, R. PRESTON, AND PHILIP J. RENY (1988): "Correlated Information and Mechanism Design," mimeo, University of Western Ontario.

MELAMUD, NAHUM, AND STEFAN REICHELSTEIN (1986): "Value of Communications in Agencies," Research Paper 895, Graduate School of Business, Stanford University.

MILGROM, PAUL R. (1985): "The Economics of Competitive Bidding: A Selective Survey," in *Social Goals and Social Organization*, ed. by L. Hurwicz, et al. Cambridge: Cambridge University Press.

MILGROM, PAUL R., AND ROBERT J. WEBER (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

MYERSON, ROGER B. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58–73.

RILEY, JOHN G., AND WILLIAM SAMUELSON (1981): "Optimal Auctions," *American Economic Review*, 71, 381–392.

[8] It is not easy to find actual markets that operate like this full-rent-extraction mechanism, in which buyers sign up in advance, knowing the pricing rule but not the seller's estimate of the item's worth. One market that seems to have this timing and commitment structure is the selling of uncut diamonds by the De Beers monopoly. Since diamonds are purchased for eventual resale, the common-value assumption fits. Each of the invited buyers is offered a single package of diamonds at a nonnegotiable price. The buyer may inspect his package; however, if he rejects it, he is never again invited to participate (Kenney and Klein (1983, pp. 500–502)).