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COSTLY SEARCH AND RECRUITING*

BY PETER HOWITT AND R. PRESTON MCAFEE[1]

This paper provides a simple model of market exchange, which, while stylized, possesses constructs that can plausibly be interpreted in terms of a typical labor market. The process of hiring a worker involves frictions, and both workers and firms may expend effort to overcome these frictions. All of the model's agents optimize. In this single framework, effects found by Diamond [1982b], Mortensen [1982], Pissarides [1984] and Negishi [1976] exist. In addition, the model is simple enough that a wide variety of market structures and institutions may be incorporated into the model with varying degrees of difficulty. These include: take it or leave it wage offers, wages determined by the Raiffa bargaining solution, advertising, variation in the disutility of work, raiding by firms of other firms' workers, word-of-mouth information transfer, variety in initial information, and some assumptions about intermediaries in the job/worker matching process. A second use of the model involves comparative statics in a multiple equilibrium world. Limited comparative statics exercises can be performed in this model, despite an inherent indeterminacy, because the set of equilibria move in a uniform way.

Recent contributions by Peter Diamond [1982b, 1984] have shown how search theory might be used to explain abnormally low levels of economic activity. The key to Diamond's analysis is an externality related to the common idea that trading is more costly the thinner the market. Specifically, the more activity there is on one side of the market the lower the costs faced by those on the other side wanting to contact a trading partner. Thus the expectation of an abnormally low level of economic activity can lead to the expectation of thin markets and high contacting costs, and thus discourage people from engaging in trading activities, thereby fulfilling the original expectation. People might all be willing to trade at higher levels, but the fact that no one else is doing so dissuades each trader from communicating a willingness to do so himself.

Diamond's contributions suggest a possible explanation for the phenomenon of persistent large-scale unemployment. However, they are cast in terms of models that are hard to relate to every day labor-market phenomena.[2] The

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[1] The authors wish to thank, without implicating, John McMillan, Hans-Werner Sinn, an anonymous referee and reader, and the participants of the Western Ontario theory workshop for useful comments.

[2] More specifically: (a) production, instead of using hired inputs through a continuous production function, consists of the random arrival at discrete points in time of opportunities

(*Continued on next page*)

present paper examines an explicit model of the labor market, in which Diamond's thin-markets externality is present. Both workers and firms actively engage in contacting activities; namely search and recruiting. Successful contacts result in lifetime wage-contracts being signed. Workers find it equally costly to search whether they are currently employed or unemployed.

The thin-markets externality is embodied in the function that depicts the contacting institutions and technology. This function gives the rate of new hiring per unit of time in the economy as a function of the number of active participants on each side of the market and of their respective intensities of contacting activity. Rather than take this function as given we derive it from an explicit description of the underlying institutions and technology.

Under appropriate conditions there exist a continuum of stationary perfect-foresight equilibria, with different wages and rates of unemployment. Also, for almost each equilibrium wage rate there exist at least two equilibrium (natural) rates of unemployment; likewise for each equilibrium value of the typical firm's recruiting intensity. All equilibria are Pareto inefficient.

The non-uniqueness obtains for two separate reasons. The first reason has to do with externalities in the recruiting process. There is the thin-markets externality whereby more recruiting by firms raises the optimal rate of job-search by unemployed workers, which in turn makes recruiting more attractive. But there is also an external diseconomy; more recruiting by firms will reduce the pool of unemployed from which the firms recruit, thereby reducing the incentive to recruit. This "common-property" externality has been analyzed by Mortenson [1982], Diamond [1982a] and Pissarides [1984]. The two externalities interact so as to yield some equilibria with active search and recruiting and others with less active search and recruiting.

The second reason has to do with a fundamental indeterminacy of wages in the model. Because of contacting costs, all bargaining will be in a situation of bilateral monopoly. The opportunity cost of signing a contract will be less than the market wage to the worker, who would have to recommence costly search to get that wage elsewhere, and greater than the market wage to the firm, who would

(*Continued*)
to produce one unit of output, at a random subjective cost to the agent; (b) each agent's ability to store goods is limited to exactly one unit, which requires him to forego any possibility of producing while attempting to sell goods that have already been produced; (c) no agent can attempt to sell goods unless he has already produced them, which, in combination with (b) comes close to an assumption that many have argued discredits search theory as a vehicle for explaining unemployment, namely that work and job-search are mutually exclusive activities; (d) it isn't clear which activity in the model ought to be interpreted as unemployment and which as employment. (Diamond interprets waiting for a production opportunity as unemployment, whereas this kind of waiting is presumably a necessary input into production and ought to be interpreted as employment. The activity of looking for a trading partner seems to us analogous to job search, but he calls it employment.); and (e) after each act of production search begins anew with no possibility of recalling the previous trading partner, in contrast to the long-term bilateral relationships typical of real-world labor markets.

have to pay extra recruiting costs to find an alternative worker. The indeterminacy of the solution to the bilateral monopoly problem induces an indeterminacy in the equilibrium market wage.

The inefficiency of equilibria is similar to the inefficiency in Diamond's [1984] model, in which the indeterminacy of bilateral monopoly is eliminated by imposing the Raiffa bargaining solution. The related work of Mortenson [1982] suggests that there might be some alternative solution that will yield an efficient equilibrium. But our results imply that in the present model no such solution exists.[3]

1. THE MODEL

The economy is populated by a large number n of identical firms, located uniformly through space, and a continuum of identical workers. There are two tradeable objects: goods and labor-services; neither of which is storeable. Firms produce goods using labor, with a constant average product, $f>0$. Firms live forever and have no endowments other than their technology. Each worker dies according to a Poisson process with a death rate of $\delta>0$; during his life he has an endowed flow of one unit of labor services per unit of time. The flow rate of new births at each instant is an exogenous constant $L>0$.

Each worker is born at a random location in space. In order to trade he must first seek to contact a firm. Until such a contact has been made no communication is possible with a potential trading partner. Until he has made contact he travels through space at the speed α. He has no prior information concerning the location of firms, and thus chooses his direction at random. Alternatively, he can choose his speed of search but not his direction. As he searches he gains no new information that would help him better to direct his search, until he finally contacts a firm. Speed is costly. The rate of disutility from search is given by the function $c(\cdot)$, which satisfies:

$$c(0) = c'(0) = 0;\ c''(\alpha) > 0 \quad \text{for all} \quad \alpha \geq 0,\ c'(\alpha) \to \infty \ \text{ as } \alpha \to \infty. \tag{1}$$

This concept of search emphasizes the importance of finding an appropriate match, which involve searching for the right trading partner rather than for the right price. Otherwise it would make little sense to suppose that the searching worker doesn't know the location of any firms. It is best to think of the model as applying to a representative "subsector" of a bigger labor market, where "false" contacts with trading partners of the wrong type are not explicitly represented.

A firm can also contribute to the contacting process, by increasing its visibility, or casting a recruiting net about itself. Consider figure 1. The firms are represented by dots, and the circles represent their recruiting nets, the places from which they are visible. When a worker runs into a recruiting net, a contact has taken

[3] Pissarides [1984] also finds that no efficient solution exists in a related model, although his efficiency criterion is a steady state criterion that ignores costs of transitions. His model embodies a more general matching technology than the present study, but lacks the treatment of multiple equilibria.

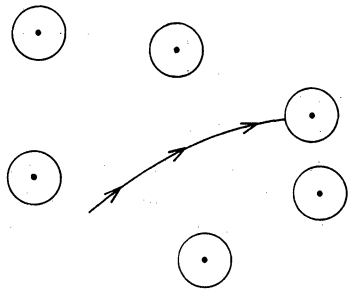

FIGURE 1
A CONTACT OCCURS WHEN A WORKER'S PATH INTERSECTS A FIRM'S NET

place. Firms can vary the size θ of their recruiting nets.[4] The cost of θ, measured in units of the consumption good, is given by the function $G(\cdot)$, which satisfies

$$G'(\theta) > 0,\ G''(\theta) > 0 \qquad \text{for all } \theta \geq 0. \tag{2}$$

This recruiting activity is the only means available to the firm for communicating with prospective trading partners. Because there is no way that a firm can send messages to a worker that has not yet fallen into its net it therefore cannot in any way influence the direction in which the workers search.

We shall restrict our attention to symmetric equilibria, in which all firms recruit at the same intensity θ and all households search at the same intensity α. In such an equilibrium each searching worker will find that the time elapsed before he contacts a firm is homogeneous of degree -1 in α, because doubling the speed halves the time to reach any given goal. In addition, because firms are identical and uniformly located, the time to meeting the first firm follows a Poisson process, with rate proportional to α.

Each recruiting firm will find its flow of contacts is proportional to the size of its net θ, because the identical searchers are uniformly located in space.[5] In

[4] For an informal example, consider workers travelling on a maze of roads. Firms place signs around, informing workers of job opportunities at the firm. If the workers travel twice as fast, they will double the number of signs they see on average. Alternatively, if the firms double the number of signs posted, this will double the number of people seeing their sign. The "recruiting net", θ, is the proportion of the roads from which a sign is visible, while the search rate, α, is the speed searchers travel. Observe that one can increase θ either by erecting more signs, or by placing them up higher.

[5] This assumes implicitly that the firms are far enough apart that their nets do not overlap. This assumption of sparsely located firms is consistent with our emphasis on the difficulties of contacting. If it were relaxed then the model would have to deal with a congestion externality, as one firm's increase in θ would, beyond some point, reduce the hiring by other firms whose area it was invading. Such externalities would obviously be important for the present analysis. However, as we have observed, there is already a dynamic version of that externality (the common-property externality) in the present model, which becomes increasingly important as θ increases. (More specifically, that externality is the reason why the marginal benefit of recruiting, as shown in Figure 3, eventually falls below the marginal cost.)

fact it will find this flow proportional to $\alpha\theta u$, where u is the number of searching workers, because doubling any one of these variables doubles the rate of contacting. Furthermore, because there is a continuum of workers per firm, this flow is deterministic. In addition, the workers obtain contacts at a rate proportional to $\alpha n\theta$, and by choosing appropriate units for time and θ, we may make both constants of proportionality equal to unity.

It follows that the aggregate rate of contacting will be equal to the product $\alpha\theta nu$. This contacting technology embodies the thin-markets external economy discussed earlier. The marginal product of the speed of search α in producing contacts is $n\theta u$, which is proportional to the recruiting intensity θ; likewise the marginal product of recruiting intensity is proportional to the speed of search. The technology also embodies the "common-property" diseconomy. A reduction in the number of searchers u will lower the marginal product of the firms' recruiting intensity.[6]

2. WAGE BARGAINS

Each contact will result in the firm and worker agreeing to a lifetime contract according to which the worker will supply his services until he dies, at a negotiated wage payed in the form of goods. We shall suppose that all negotiations will result in a constant wage over the lifetime of the match, and that each contract results in the same wage $w>0$. For simplicity we assume the household has no value of leisure.[7] Thus as long as w is less than f, there will be no quitting or firing once a contract has been made. This is because a worker's best alternative upon quitting would be to recommence a costly search for the same contract, and a firm who fires a worker will be giving up the flow of net marginal products $f-w>0$ from that worker. Likewise there will be no on-the-job search because it is costly and cannot result in a higher wage.

As we have already mentioned, each wage-negotiation takes place in a situation of bilateral monopoly. Furthermore we assume that each agent is risk-neutral[8],

[6] There are other, potentially important, labor-market externalities that we do not consider. For example, an increase in the flow of applicants to a firm with given net size might increase his costs by making him have to sort through more applications. Likewise, an increase in the flow of applicants might raise the cost to any given applicant by making him wait longer in line. We rule out these phenomena by assuming that all transaction costs are those implied by the spatial considerations that we wish to focus on. Once a contact has been made there are no further transaction costs.

[7] This implies that the search cost c is an effort cost, not a foregone leisure cost. Our earlier [1984] paper deals with the case where leisure is valuable and must be given up entirely if the worker wishes either to search or to work; the main results of that paper were the same as of the present paper with the exception that workers faced a non-trivial decision whether or not to enter the labor force.

[8] That is, his utility is linear in the flow of real income at each date. Assumption (1) implies that a worker is averse to risk in the speed of search, but search is a bygone when a contract is being negotiated.

with the same rate of time preference r. The total expected discounted utility to be divided by the bargainers is $\frac{f}{r+\delta}$. Any contract that resulted in no quitting or firing would yield this total. Any other contract would yield a strictly smaller total. If the two parties agree to a contract at the market wage there will be no quitting or firing and hence the agreement will be privately efficient. Thus, any wage in the interval $(0, f)$ will be privately efficient.

In the same situation there will be many alternative agreements that would also be privately efficient. Because of this it seems that other equilibria (with wages varying across contracts and with contracts specifying a time-varying wage) will probably exist. Because we get a surfeit of symmetric constant-wage equilibria we are not interested in these other equilibria.

3. THE WORKER'S DECISION PROBLEM

In the interest of simplicity we shall restrict attention to stationary equilibria, in which not only w but also α, u, and θ are constant. The individual worker makes no choices if he is employed. If he is unemployed his only decision is to choose the optimal speed α with which to look for a job. He makes this choice knowing the market value of w and θ.

If he makes a contact he will receive a contract worth $\frac{w}{r+\delta}$ in expected discounted utility. The probability of making such a contact over the next instant is $\alpha n\theta$. Thus V, the value of searching, must satisfy the equation:

$$rV = \alpha n\theta\left(\frac{w}{r+\delta} - V\right) - \delta V - c(\alpha)$$

or:

$$V = \frac{\alpha n\theta\left(\frac{w}{r+\delta}\right) - c(\alpha)}{r+\delta+\alpha n\theta}. \tag{3}$$

Equation (3) has a straightforward interpretation. The value of a "ticket to search" is the discounted value of the flow of expected net returns to the ticket. The flow is the expected payoff per unit time $\alpha n\theta\left(\frac{w}{r+\delta}\right)$ minus the cost $c(\alpha)$. The appropriate discount factor includes not just the rate of time preference but also the rate at which the ticket can be expected to "depreciate" because of search-termination due to death or contact, $\delta+\alpha n\theta$.

The searching worker chooses α to maximize (3). The first-order condition for this problem is:

$$c'(\alpha) = n\theta(w+c(\alpha))/(r+\delta+\alpha n\theta). \tag{4}$$

It follows easily from (1) that a unique optimum exists to this problem, which we denote by the function $\alpha^*(w, \theta)$, and that,

$$
(5)\quad
\begin{aligned}
&\text{(a)}\quad \alpha^*(0,\theta)=\alpha^*(w,0)=0 \qquad \forall w,\theta\geq 0\\
&\text{(b)}\quad \alpha^*(w,\theta)>0 \quad \forall w,\theta>0\\
&\text{(c)}\quad \alpha^*_w(w,\theta)=1\Big/\left(\frac{r+\delta}{n\theta}+\alpha\right)c''(\alpha)>0 \quad \forall w\geq 0,\theta>0\\
&\text{(d)}\quad \alpha^*_\theta(w,\theta)=c'(\alpha)(r+\delta)/(r+\delta+\alpha n\theta)\theta c''(\alpha)>0 \quad \forall w>0,\theta\geq 0\\
&\text{(e)}\quad \lim_{w\to\infty}\alpha^*(w,\theta)=\infty \qquad \text{for all} \quad \theta>0.
\end{aligned}
$$

The result 5(d) shows one aspect of the thin-markets externality. An increase in recruiting intensity will induce workers to increase their speed of search even with no change in the wage.[9]

The number of searchers u is the number of unemployed young people who have yet to make their first contact. The flow of people into the pool of unemployed is the rate of birth of new workers L. The flow out is the flow of deaths δu plus the flow of new contacts: $\alpha n\theta u$. In a stationary equilibrium, the inflow and outflow must be equal. Therefore the number of unemployed searchers will be given by the function:

$$
(6)\qquad u=u^*(w,\theta)=L/(\delta+\alpha^*(w,\theta)n\theta)\,.
$$

4. THE FIRM'S DECISION PROBLEM

Each firm must decide its recruiting intensity knowing that there are u workers each searching at the rate α, and knowing that each contact will result in the wage w. The firm takes α as given, even though according to (4) α depends upon θ, because it conjectures that a unilateral increase in its own individual recruiting intensity would be too insignificant to influence household search behavior. Likewise it takes u as given. It takes w as given because it rationally anticipates that this wage is the best it can get from any negotiation.

The expected discounted utility attributable to the firm's current recruiting activities is $\theta\alpha u\left(\frac{f-w}{r+\delta}\right)-G(\theta)$ where $\theta\alpha u$ is the flow of hiring and $\frac{f-w}{r+\delta}$ is the expected present value to the firm of each new hire. It chooses θ so as to maximize this amount; i.e. so that:

$$
(7)\qquad G'(\theta)-\alpha u\left(\frac{f-w}{r+\delta}\right)\geqq 0, \text{ with equality unless } \theta=0.
$$

It follows from (2) that for any $u>0$ and any $w\in[0,f]$ there is a unique solution $\theta^*(\alpha,u,w)$ to this problem. The dependence of θ^* on α again reflects the thin-market externality. According to (6) and (7) θ affects and is in turn affected by u. This is the common-property externality.

[9] To verify 5(e) note that if $w\to\infty$ and α remains finite then the right-hand side of (4) will eventually exceed the left hand side.

5. EQUILIBRIUM

An equilibrium is a symmetric, stationary situation in which each firm chooses the same $\theta > 0$, each household chooses the same $\alpha > 0$ and the level of u is constant. By substituting for α and u in (7) using (4) and (6) we can define it equivalently as a pair (w, θ), with $w \in (0, f)$ and $\theta > 0$, such that:

$$G'(\theta) = \sigma(w, \theta)\left(\frac{f-w}{r+\delta}\right) \tag{8}$$

where $\sigma(w, \theta)$ is the arrival rate of workers to a firm, per unit of θ:

$$\begin{aligned} \sigma(w, \theta) &\equiv \alpha^*(w, \theta)u^*(w, \theta) \\ &= \alpha^*(w, \theta)L/(\delta + \alpha^*(w, \theta)n\theta). \end{aligned} \tag{9}$$

Equation (8) just equates the marginal cost and benefit from recruiting, where the marginal benefit itself depends upon the recruiting intensity through effects that are external to the firm choosing θ; i.e. through its effects on α^* and u^*.

It is easy to derive conditions under which an equilibrium will exist.[10] In general when one equilibrium exists a continuum exists, because it is defined as a pair of variables satisfying a single equation. This indeterminacy is a result of the bilateral monopoly indeterminacy.[11]

Furthermore, the set of equilibria will generally consist of closed loops, as in Figure 2, because for every equilibrium with a given wage there will exist another with the same wage. Also, for every equilibrium with a given recruiting intensity there will generally exist another with the same recruiting intensity.

The reason for these closed loops is the interaction of the externalities. As Figure 3 illustrates, when θ falls towards zero so does the marginal benefit from recruiting, because the fall in θ induces households to slow down their search through the thin-market externality, which makes the arrival rate σ fall towards zero (recall (5a) and (9)). Because of (2) the marginal cost of recruiting does not fall all the way to zero. Therefore for any given w and small enough θ marginal cost will exceed marginal benefit. But the same will be true for large enough θ, because as θ goes to infinity the common-property external diseconomy makes the number of searchers and hence the arrival rate σ fall to zero (note that (5b) and (9) imply $\sigma < L/n\theta$). Thus if marginal benefit equals marginal cost for some (w, θ) it will generally do so for at least one other θ and the same w. As

[10] See Howitt and McAfee [1984].

[11] This indeterminacy might be taken to imply nothing more than that we have an incompletely specified model. We do, however, have a complete description of tastes and technology, the classical determinants of economic variables. Thus, one can also take our interminacy to imply that the classical determinants are not sufficient in a world such as the one we have described. In section 8 below we discuss how to eliminate the indeterminacy by imposing various bargaining solutions. Even without imposing such a solution we show in Section 7 how our "incomplete" model yields meaningful comparative-statics propositions.

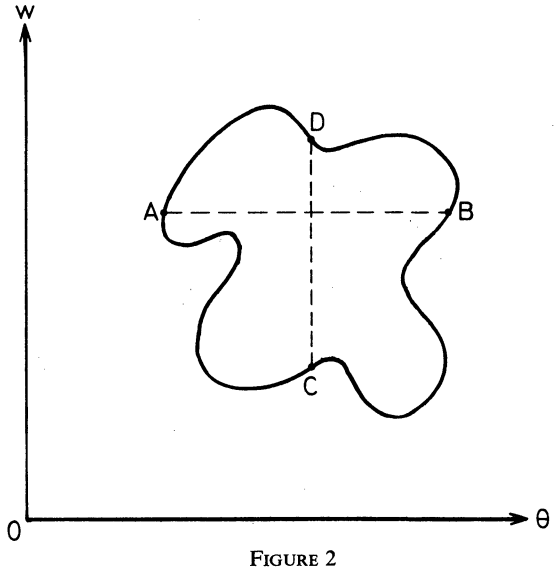

FIGURE 2
A CLOSED LOOP OF EQUILIBRIA

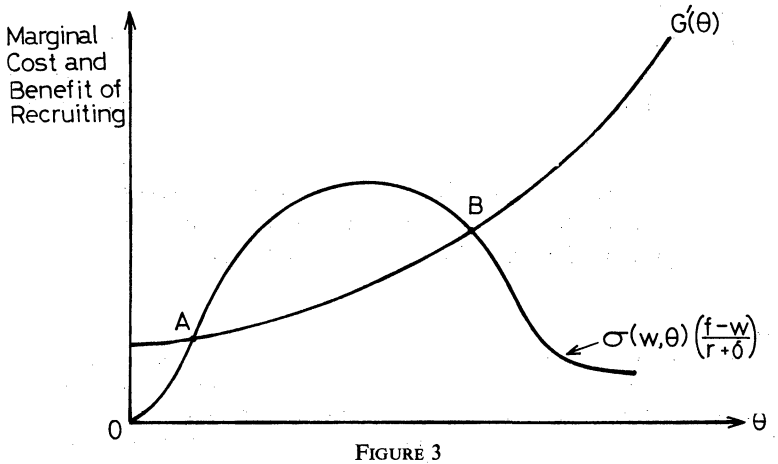

FIGURE 3
MULTIPLE EQUILIBRIUM VALUES OF θ

in Diamond's models there will be equilibria with a lot of activity (θ) and others with not so much.[12]

[12] One might wonder how the economy could get started from an initial position of zero employment, since no firm will find it worthwhile to start recruiting until others start recruiting. Actually this positive interdependence of recruiting decisions poses a problem no matter what

(*Continued on next page*)

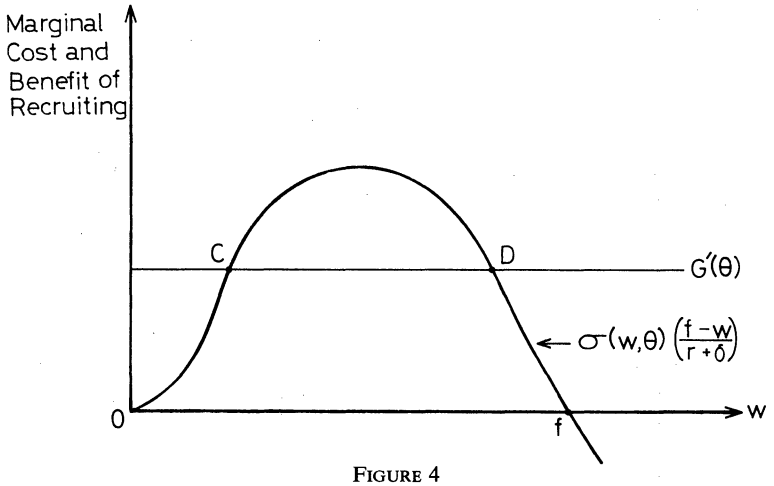

FIGURE 4
MULTIPLE EQUILIBRIUM VALUES OF w

Likewise, as Figure 4 shows, if we hold θ fixed then the marginal benefit will fall below the marginal cost for wage rates that are too high, or too low. High wages reduce the incentive to hire for obvious reasons. Low wages reduce it because they discourage search, and that reduces the arrival rate σ.

Note that the result of multiple values of θ for a given w depends upon our assumption that $G'(0)>0$. This assumption could easily be replaced with others. For example, if $G'(0)=0$ but $G''(0)$ is greater than $\sigma_\theta(w, 0)\left(\frac{f-w}{r+\delta}\right)$ then the same reasoning goes through. Alternatively, as in an earlier version of the paper [1984] we could assume that there is a distribution of entry costs across households that makes the participation rate fall continuously to zero as θ falls to a positive

(Continued)
the initial position. Even if the economy had been forever in a stationary equilibrium with positive employment, if everyone suddenly formed the belief that θ would equal zero from now on, that would be a self-fulfilling belief. No worker would search, no firm would recruit, and employment would fall to zero through attrition at the rate δ. This alerts us to the fact that starting at any given initial employment level there may be more than one perfect-foresight equilibrium path. Starting from a non-degenerate stationary equilibrium one such path stays there and another converges on zero employment. There is more than one set of self-fulfilling beliefs. But by the same token if we start at zero employment there will generally exist paths that converge to a "high-level" equilibrium. Remember that starting with low employment is itself an inducement to high recruiting because it lowers the cost of hiring. The dynamic version of this model presented in our earlier [1984] paper shows that if the wage is fixed then there will generally exist an equilibrium path starting at zero employment, along which θ *exceeds* the highest level stationary equilibrium value $\hat{\theta}$ associated with the fixed wage, and falls asymptotically to $\hat{\theta}$ as employment rises to its stationary value. The phenomenon of multiple stationary solutions in similar models is examined more fully in our [1986] paper.

lower limit $\bar{\theta}$ at which the optimized value of V has fallen to equal the lowest entry cost. At that point, the arrival rate of searchers to a firm will fall to zero, and hence the marginal benefit of recruiting will fall to zero.

The indeterminacy of equilibrium implies an indeterminacy of the rate of unemployment. The steady-state labor force is L/δ. Thus, the rate of unemployment is $u^*(w, \theta)\delta/L$, which is decreasing in both w and θ. As you move up an upward-sloping part of the closed loop the rate of unemployment will be continuously decreasing.

6. PARETO INEFFICIENCY OF EQUILIBRIA

Because of the externalities, equilibria are inefficient. This can be shown in two different senses. First, consider as welfare criteria the expected utility of the unborn worker and the expected discounted profits of each firm. It follows from (3) and (4) that the former equals:

$$V^*(w, \theta) = \frac{1}{r+\delta}\int_0^{\alpha^*(w,\theta)} xc''(x)dx$$

which is increasing in w and θ. The latter is

$$((f-w)(L/\delta-u^*(w, \theta))/n - G(\theta))/r$$

where $L/\delta-u^*(w, \theta)$ is total employment (labor force minus those employed). By (9), total employment also equals $n\theta\sigma(w, \theta)/\delta$. Thus by (8), the firm's expected profits can also be expressed as:

$$W^*(\theta) = ((r+\delta)\theta G'(\theta)/\delta - G(\theta))/r$$

which, by (2) is a strictly increasing function of θ.

It follows that some equilibria Pareto-dominate others in terms of these criteria. Specifically, as you move to equilibria with higher w and/or higher θ expected utility of workers goes up and that of firms does not go down.

This concept of inefficiency ignores any transitional gains or losses in moving from one stationary state to another. But the following argument shows that even taking these dynamic considerations into account all equilibria are inefficient.

Consider the social problem of maximizing the discounted sum of total utility of all workers and firms:

$$\underset{\{\alpha,\theta,u\}}{\text{Max}} \int_0^\infty e^{-rt}\{f(L/\delta-u) - uc(\alpha) - nG(\theta)\}dt$$

subject to $\dot{u} = L-(\delta+\alpha n\theta)u$
given an initial value of unemployment. The necessary conditions to this problem are:

$$-uc'(\alpha) - \theta nu\lambda = 0$$

$$-nG'(\theta) - \alpha nu\lambda = 0$$

$$\dot{\lambda} = r\lambda + (f+c(\alpha)) + (\delta+\alpha n\theta)\lambda\,.$$

Thus if u, α, θ provide a stationary solution to the problem they must satisfy:

$$c'(\alpha) = \theta n(f+c(\alpha))/(r+\delta+\alpha n\theta) \tag{10}$$

$$G'(\theta) = \alpha u(f+c(\alpha))/(r+\delta+\alpha n\theta)\,. \tag{11}$$

Comparison of (10) and (4) reveals that the equilibrium can coincide with this stationary social optimum only if $w=f$. But, by the private optimality condition (7) and condition (2) the equilibrium must have $w<f$. Thus there is no wage compatible with an equilibrium that maximizes the sum of discounted utilities. In this sense, all equilibria are Pareto-inefficient.

The economic interpretation of this last result is straightforward. The efficient speed of search is one that equates the marginal cost $c'(\alpha)$ to the marginal social benefit. The latter is the marginal product of α in producing contacts, $n\theta$, times the social value of a contact. But the only way to induce a household to search that intensively is to promise him the entire social value f, if he makes a contact, i.e. to pay him a wage equal to his marginal product. But if firms must pay a wage equal to the marginal product there is no gain to recruiting and hence they will not recruit.

Mortenson [1982] found that there was an efficient wage in a related model. The key difference in this model preventing the existence of such a wage is that the contacting rate $\alpha n\theta u$ exhibits increasing returns in the contacting intensities α and θ, whereas in Mortenson's model there were constant returns in the intensities α and θ. An efficient wage would be one that paid contacting activities at rates equal to their respective marginal value products. With constant returns such payments would be possible. But with increasing returns they would more than exhaust the economy's total output.

7. COMPARATIVE STATICS RESULTS

Even with all this indeterminacy our model yields comparative statics predictions if we are willing to treat the wage and recruiting intensity as exogenous variables. Thus, for example, (6) implies that a ceteris paribus increase in w or θ will reduce the equilibrium number unemployed. By the same token such a change will reduce the equilibrium rate of unemployment, which is just $u^*(w, \theta)\delta/L$, and will also reduce the equilibrium value of the average duration of unemployment, which is just $1/(\alpha^*(w, \theta)n\theta+\delta)$, and increase the equilibrium level of employment $L/\delta-u^*(w, \theta)$. The admissibility of such ceteris paribus changes is guaranteed by our earlier result that equilibria came in closed loops, although they must generally be discrete changes. Analogous results for continuous variations in w and θ can be derived under the assumption that w and θ vary together. Thus as w and θ rise together along an upward sloping part of a closed loop, with no other exogenous variable changing, the equilibrium values of the number unemployed, the rate of unemployment and the duration of unemployment will all decrease while equilibrium employment increases.

Comparative-statics effects of changes in other exogenous variables can be derived in the following way. An increase in, say, the marginal product of labor, will cause each closed loop of equilibria to expand outward. This can be seen from Figures 3 and 4. The rise in f will shift the marginal benefit curve up in both diagrams, causing the larger value of θ (resp. w) to increase and the smaller one to decrease. Suppose we hold w fixed. Then whether the equilibrium recruiting intensity increases or decreases depends upon which side of the loop the equilibrium is on.

Assume that:

$$\frac{\alpha c''(\alpha)}{c'(\alpha)} \cdot \frac{\theta G''(\theta)}{G'(\theta)} > 1 \quad \forall \theta, \alpha . \tag{12}$$

Condition (12) asserts that the increasing returns in contacting that come about from the fact that the rate of contacting is proportional to $\alpha\theta$ are outweighed by the increasing marginal cost of producing the intensities α and θ, in the sense that the solution to the social welfare maximization problem of the preceding section does not involve making α and θ infinite for an instant. More precisely, (12) is easily seen to be a necessary second-order condition for α and θ to provide an interior maximum to the Hamiltonian of that problem.

Under condition (12) our earlier paper [1984] shows that all points on the left-hand face of the closed loop are unstable. Intuitively this is because at point A in Figures 2 and 3 an increase in θ by all other firms would induce a firm to raise its own θ by even more. Thus if we restrict attention to stable equilibria the rise in f can be seen to cause a rise in the equilibrium value of θ. Because of this and our previous comparative-statics results it also causes a decrease in the equilibrium number of unemployed.

By similar reasoning it can be shown that the same effects on θ and u would be *produced by a reduction in the marginal cost of recruiting; i.e. by a shift from the* cost function $G(\theta)$ to the function $\hat{G}(\theta)=\gamma G(\theta)$ where $0<\gamma<1$. Likewise for an analogously defined reduction in the marginal cost of search, which can be shown to cause an increase in $\alpha^*(w, \theta)$ for given values of (w, θ) and thus, according to (9), shifts the marginal benefit curve up in Figure 3. It follows immediately that any of these changes would also decrease the equilibrium rate and duration of unemployment and increase equilibrium employment.

None of these results is surprising or difficult to derive. Nevertheless, they show that the model is capable of providing a simple account of several important labor-market variables while yielding falsifiable implications that are not obviously false.

8. VARYING THE ASSUMPTIONS

In this section, we consider how our results would be affected by altering some key assumptions. First, we consider applying different solution concepts to the bilateral monopoly situation faced by each matched pair. The concept we

consider initially is the Raiffa bargaining solution used by Diamond [1984], which splits the gains from trade evenly between the two parties. With this solution, workers and firms will still take the equilibrium wage as given when choosing their contacting intensities; thus equilibria will still have to solve the condition (8) defining the closed loops. But the requirement that the wage split the gains from trade evenly will impose a second condition on w and θ.

The gain to the worker is the difference between the expected discounted value of his lifetime wage $\frac{w}{r+\delta}$ and the value of recommencing search at the optimal rate. By using the first-order condition (4) to substitute for $c(\alpha)$ in (3) we can write this difference as $c'(\alpha^*(w, \theta))/n\theta$. The gain to the firm is $(f-w)/(r+\delta)$. The condition that these two gains be equal describes a positively sloped arc in (w, θ) space. Equilibria will consist of the points of intersection of this arc with the closed loops.

Thus the Raiffa solution generally reduces our continuum of equilibria to a discrete set. There will generally be an even number of equilibria, as in our earlier model with any fixed wage. Our previous welfare and comparative-statics results will still apply. Furthermore if we generalize the solution to allow the gains to be split in fixed proportions other than one to one, the result will be qualitatively the same. As we vary the proportion going to the workers zero to one, the upward sloping arc will vary continuously and the set of equilibria will trace out the original closed loops.

One complication is added by these solution concepts, however; workers might engage in on-the-job search, knowing that they can attract a higher wage from the second firm they contact. This is because the worker's best alternative to concluding a bargain will be better in his second contact than it was in his first; in the second contact he can always return to working at the market wage with no additional search cost. Unless this is ruled out by supposing, for example, that there is no way that a second-time searcher can distinguish himself in the bargaining process from a first-time searcher, or that search is more costly on the job than off, our single-wage equilibria are no longer equilibria under one of these bargaining solutions. Of course, a policy by the firm of matching best offers would further encourage this on-the-job search.

Another solution to the bargaining problem is that firms might make take-it-or-leave-it wage offers to newly contacted workers. Under the assumption that all offers must take the form of a constant life-time wage, workers will search as before, and the first-order condition (7) will still govern the firm's choice of θ. Thus equation (8) will still have to be satisfied in equilibrium. But now the market wage will also have to satisfy the condition that the typical firm not find it optimal to offer some other wage.

Suppose a firm decides to offer Δ below the market wage. Then its wage bill will be reduced. But its newly hired workers will engage in on-the-job search, at the rate $\alpha^*(\Delta, \theta)$, and hence will quit at the rate $\alpha^*(\Delta, \theta)n\theta$ at which they encounter other firms. Workers will continue to arrive at the same rate $\sigma(w, \theta)$

as before because the individual firm's wage-offers cannot be made until a worker has been contacted, by which time it is too late to influence his search behaviour.

If a firm decides to offer above the market wage the only effect on its profits will be to raise its wage bill. The quit rate of its employees will not be affected because that is already zero.

Thus the firm's profits from its current recruiting activities will be:

$$(13)\qquad \begin{cases} \theta\sigma(w,\theta)(f-w+\Delta)/(r+\delta+\alpha^*(\Delta,\theta)n\theta)-G(\theta) & \text{if } 0\leqq\Delta\leqq w \\ \theta\sigma(w,\theta)(f-w+\Delta)/(r+\delta)-G(\theta) & \text{if } \Delta<0. \end{cases}$$

It follows from 5(c) that this function always has a kink at $\Delta=0$, reflecting the asymmetric reaction of quits to an increase or decrease in the wage offer. Furthermore, it is strictly increasing for $\Delta\leqq 0$. Thus the firm will never offer above the market wage.

Assume that $c'''(\alpha)\geq 0$ for all $\alpha\geq 0$. Then it can be shown using (5) that the profit function (13) is convex in Δ on the interval $[0, w]$. Thus the firm will choose either $\Delta=0$ or $\Delta=w$, and the further condition on the market wage is that:

$$(14)\qquad \frac{f-w}{r+\delta}\geq\frac{f}{r+\delta+\alpha^*(w,\theta)n\theta}.$$

This condition requires (w, θ) to lie to the right of an arc that goes from (0, 0) to (∞, f) with a strictly increasing wage (see Figure 5). Equilibria consist of the intersection of this set with the closed loops. If any exist there will generally be a continuum. Again, our previous welfare and comparative-statics results will go through.

Equilibrium can be shown to exist under this solution concept if, for example, the marginal product of labor is large enough [Howitt and McAfee, 1984]. However, this existence is precarious. For if (1) were modified to assert that $c'(0)>0$ no equilibrium could exist. It would always pay a firm to offer below the market wage because a small differential will not induce on-the-job search.[13] Likewise, even if $c'(0)=0$ no equilibrium would exist if firms were allowed to offer non-constant wage profiles. It would pay a firm to offer a profile of zero for an initial period of length ε then equal to the market profile with a fixed delay of ε.

The gain to the firm in the form of a reduced wage bill per new recruit would be of the order ε. But the cost would be of the order ε^2 because the new recruits would search at a slow speed for a short time. Thus some such undercutting of the market profile would always be optimal.[14]

We now consider the question of how robust our results are to modifications that would relax the very severe limitations we have imposed on agents' ability to communicate with one another.

It seems that a limited form of price-advertising could be introduced at little cost. Consider the take-it-or-leave-it version of the model. Suppose that a firm that

[13] A similar argument for non-existence has been proposed by Stiglitz [1979].

[14] A formal demonstration of this result is given by Howitt and McAfee [1984].

offers above the market wage (i.e. a negative Δ) finds it easier to cast a net of given size because news of the extraordinary wage travels by word of mouth. Specifically, let the recruiting cost be given by the convex function $G(\Delta, \theta)$, with G_1, $G_2 > 0$. Then equilibria will still have to solve equation (8) with $G_2(0, \theta)$ replacing $G'(\theta)$, and inequality (14). But they will also have to satisfy the condition that lowering Δ below zero cannot raise the firm's profits. By the convexity of G this condition is that:

$$G_1(0, \theta) \leq \frac{\sigma(w, \theta)\theta}{r+\delta}.$$

Suppose that increasing the net size reduces the marginal cost of lowering the wage; i.e. that $G_{12} < 0$. Then solutions to this inequality are describable as the set of points lying to the left of a downward-sloping line. As shown in Figure 5 this means that equilibria, if they exist, still occur in continua, with no substantive change in our results. Economically, some indeterminacy remains in this model because allowing advertising does not remove the asymmetric quitting behaviour that kinks the profit function. This kink produces indeterminacy of the equilibrium wage for the same reason as in the models of Negishi [1976] and Woglom [1982].

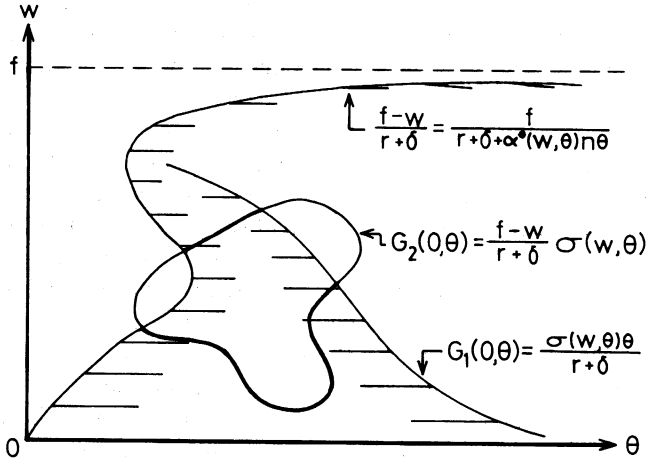

FIGURE 5
EQUILIBRIA WITH WAGE-ADVERTISING

It might also seem reasonable to allow agents to begin life with more information than they have here; to know the location, for example, of at least some firms. In this case each searcher would head straight for the nearest known firm, at a speed that is optimal given the known wage, the known distance to the firm, and the known probability of encountering a closer firm on the way. One equilibrium in this model would be $(\theta, w) = (0, f)$. The probability of encountering an

unknown firm would be zero. Households would travel at a speed that was socially optimal for the closest known firm, given that $\theta=0$. But if each worker knew few enough locations this equilibrium would not be efficient; it would be socially worthwhile to have some recruiting to catch workers headed to more distant firms. Likewise, unless households began by knowing all the locations the externalities that made equilibria in our earlier model come in closed loops would still exist. A bigger θ would induce faster search, which would tend to make recruiting more profitable up to some point, but would also deplete the stock of searchers and therefore tend to make recruiting less profitable. Likewise the same bilateral monopoly problem would continue to make the equilibrium indeterminate.

The implication of our model that all search is pointless if $\theta=0$ seems overly restrictive. Surely a diligent enough job-searcher should be able to communicate an offer to a firm that is not actively in the market. This possibility could be allowed for by setting $G(\theta)=0$ for all θ in some interval $[0, \tilde{\theta}]$. Thus all firms would always have a net size of at least $\tilde{\theta}$, and a searcher's probability of encountering a firm would never fall to zero unless he stopped moving. All that this would affect in our analysis is the result that at least two equilibrium values of θ will exist for each equilibrium w. As Figure 3 shows that result would no longer follow. But it is equally clear from Figure 3 that the result would still be possible; it would just require the marginal benefit and cost curves to interwine in the region where marginal benefit was increasing. Furthermore, the result would follow by necessity if we also added the assumption discussed earlier that there is a distribution of participation costs with a positive lower bound. For at least some values of the wage, the value $\bar{\theta}$ at which the optimized value of search equals the lowest participation cost would be greater than $\tilde{\theta}$, and hence the marginal benefit curve would first intersect $G'(\theta)$ from below in Figure 3, yielding more than one equilibrium θ.

Another implication that ought to be relaxed is that of no raiding. Surely it ought to be less costly for a firm to recruit by sending messages to the employees of other firms, whose locations can be identified, than by sending them aimlessly into space. If we allowed raiding that was costless to the raiding firm it seems that equilibrium would no longer exist in our model. No firm would cast a recruiting net. Instead they would send offers to employees of existing firms. The only equilibrium offer would be a wage equal to the marginal product. But with no recruiting nets no contacts would ever get made.

However, as long as raiding were costly we would get the same equilibria as before, provided that we allowed firms to adopt a policy of matching any offers their employees received from a raiding firm (which they would find optimal to do) and provided that in the case of a matched offer the employee always decided to remain. Under these assumptions no raiding would ever be attempted, because it would produce positive costs and no benefits.

A final restrictive assumption is that of no intermediaries. If everyone could costlessly communicate with one of a number of competitive intermediaries then

the model would have no indeterminacy and no externalities, because people would, in effect, be costlessly communicating with one another, and all remaining costs of hiring would be internalized by the intermediaries. What would happen with intermediaries that were costly to find, and who themselves had to advertise their location, would require a more complicated model to deal with. But as long as the communication costs remained so would the basis for our major results; the contacting intensities of the intermediaries would interact with those of the primary agents in the same qualitative manner as do the search and recruiting intensities in the present model.

Intermediation could solve the externalities problems if there were private ownership of the space through which workers search and firms broadcast. For example, a single owner could charge an entry fee to firms and households. These entry fees could be set high enough to appropriate all the economy's surplus. In order to maximize its profit the intermediary would promise households a rebate of $\frac{f-w}{r+\delta}$ for having found a job, and it would rebate to firms the amount $\frac{c'(\hat{\alpha})}{n\hat{\theta}} - \frac{f}{r+\delta}$ for each hire, where $\hat{\alpha}$ and $\hat{\theta}$ are the socially optimal values that satisfy (10) and (11). This scheme would induce workers and firms to set $\alpha=\hat{\alpha}$ and $\theta=\hat{\theta}$, resulting in a unique, socially optimal outcome.

Of course the problem with such schemes is that they would require a very large firm with a great deal of information to act as the intermediary. Such a large firm might be uneconomical for reasons of internal diseconomies. It would also be subject to misrepresentation of contacting activities by the other firms and households. And if there were differences across agents it would also be subject to misrepresentation of the agent-specific parameters that enter the fee/rebate schedule. Our own view is that one can reasonably model the labor market on the assumption this this kind of centralized organization is prohibitively costly.

9. CONCLUSION

According to Leijonhufvud [1968], persistent, large scale unemployment is best understood as a communication failure. Sellers who receive few messages of a willingness to buy will themselves be discouraged from communicating a willingness to buy, in a self-reinforcing manner. Leijonhufvud argued that these failures could not be accounted for in conventional competitive equilibrium theory where the auctioneer handled all communications problems costlessly. Diamond's recent work promises a method for making this view operational. In this paper we have tried to show that the method can be applied to models in which there are identifiable counterparts to real-world labor-market phenomena. Multiple equilibria exist with different rates of unemployment. Both high and low unemployment rates can exist at the same real wage rate for reasons that are related to the communication failures analyzed by Leijonhufvud; the expectation

of a low level of labor-market communication can be self-fulfilling. Although barriers to communication are at the heart of these results we have argued that the specific barriers assumed in the present paper could be relaxed considerably without changing the essential message of the analysis.

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