CALTECH

Ec 11 Midterm Examination
Professor R. Preston McAfee
Spring 2006

Instructions: Open book, open notes, no collaboration.
Partial credit will be assigned. Please show your work.
You may take this test during any consecutive 4 hour period.
Due May 2, by 5:00 PM. Please deposit in Box outside Baxter 100.

1. (20 points) The short run elasticity of demand for airline tickets is 0.1, and the long run demand elasticity is 2.4. The cost of a ticket from Los Angeles to New York was $400, and 10,000 tickets were sold each week. New fees were just introduced, causing prices to increase by $50.
   a. Assuming a perfectly elastic short run supply, how many tickets are expected to be sold this week?
   b. Why is the demand elasticity so different in the short run and the long run?

2. (20 points) Each automated doorknob factory costs an amount with present value equal to $1,000,000 per year to build, and produces $\frac{1}{2}\sqrt{x}$ thousand doorknobs per year, where *x* is the input of materials in tons.
   a. If the price of materials is $4/ton, find the long and short run industry supply of doorknobs.
   b. Does the short run supply of doorknobs have constant elasticity?
   c. Starting with the doorknobs market in long run equilibrium, and a demand elasticity of one, compute the short run effect on price of a decrease in the price of materials to $1/ton.

3. (20 points) Consider the production possibilities frontier for wheat *W* and corn *C*. Nation one has a frontier of $W^2 + 3C^2 = 4$ and nation two has a frontier of $2W^2 + 6C^2 = 4$. Both nations consume equal amounts of corn and wheat.
   a. Under autarky, how much does each nation consume?
   b. What is the marginal cost of corn of each nation?
   c. If the relative price of corn to wheat is 1, will trade occur between the two nations?

4. (10 points) Explain why the supply of labor can be described as "backward bending."

5. (20 points) Safety regulations require a factory to have 3 workers (y) for every machine (x) it runs. The factory's output is given by $T = 15x^{0.5}y^{0.2}$. If the price of x is p, and the price of y is r, what ratio of p to r would make the required input mix optimal?

6. (10 points) How would you use economic analysis to assess whether movie theater tickets and movie DVDs are complements or substitutes in demand? Name two products outside of the movie industry that may be affected by an increase in the price of movie tickets, and explain how.

Answers

1. (20 points) The short run elasticity of demand for airline tickets is 0.1, and the long run demand elasticity is 2.4. The cost of a ticket from Los Angeles to New York was $400, and 10,000 tickets were sold each week. New fees were just introduced, causing prices to increase by $50.

   a. Assuming a perfectly inelastic short run supply, how many tickets are expected to be sold this week?
   b. Why is the demand elasticity so different in the short run and the long run?

a. With constant elasticity, $q = ap^{-0.1}$, so $10000 = a(400)^{-0.1}$ and $a = 10000(400)^{0.1}$. Thus, after the fare increase, $q = a(450)^{-0.1} = 10000\left(\frac{400}{450}\right)^{0.1} = 9883$. Will also accept the answer that a 12.5% increase should yield approximately a 1.25% decrease in quantity, which would result in a quantity of 9875.

b. In the short run, consumers cannot readily reschedule business meetings, change vacation plans, or move to an area that requires less flying. In the long run, they could make these adjustments and find alternatives to air travel, so demand is more elastic.

2. (20 points) Each automated doorknob factory costs an amount with present value equal to $1,000,000 per year to build, and produces $\frac{1}{2}\sqrt{x}$ thousand doorknobs per year, where *x* is the input of materials in tons.

   a. If the price of materials is $4/ton, find the long and short run industry supply of doorknobs.
   b. Does the short run supply of doorknobs have constant elasticity?
   c. Starting with the doorknobs market in long run equilibrium, and a demand elasticity of one, compute the short run effect on price of a decrease in the price of materials to $1/ton.

Answer: Let *c* be the cost of materials per ton, and M a million. If *y* is the output of doorknobs in thousands, $y = \frac{\sqrt{x}}{2}$ or $x=4y^2$, so the total cost of *y* is M + $4cy^2$. Let *p* be the price. The short run supply of any given plant equates *p* and marginal cost, so *p* = 8*cy*, or $y = \frac{p}{8c}$. If there are *N* factories, short run supply is $y = \frac{Np}{8c}$. Minimum average variable cost is zero so there is no shutdown point. The long run supply minimizes average total cost, which is $\frac{\text{M} + 4\text{cy}^2}{y} = \frac{\text{M}}{y} + 4\text{cy}$. This is minimized when $0 = -\frac{\text{M}}{y^2} + 4\text{c}$, or $y = \sqrt{\frac{\text{M}}{4\text{c}}} = \frac{500}{\sqrt{c}}$. Minimum average total cost at this quantity is $\min ATC = \frac{\text{M}}{\sqrt{\frac{M}{4c}}} + 4\text{c}\sqrt{\frac{M}{4c}} = 2\sqrt{4Mc} = 4000\sqrt{c}$, per thousand doorknobs.

a. The value of *c* is 4 the above, for an average cost of $8,000 per thousand doorknobs.

b. Yes, since it has the form $s(p) = ap^{\eta}$. (The elasticity is 1, and a=1/8*c*).

c. In long run equilibrium, the price is minimum average total cost, which is $4000\sqrt{c} = 8000$. The long run price falls to $4000\sqrt{1} = 4000$. Let there be *N* factories initially. Prior to the cost drop, the quantity supplied equaled the quantity demanded, and $y = \frac{Np}{32}$. After the cost decrease, *c* is 1, sp the short run supply is $y = \frac{Np}{8}$, which is four times the old short run supply. The demand comes in the form $q = ap^{-1}$. Prior to the decrease, short run equilibrium implies quantity demanded equals quantity supplied, or $\frac{N8000}{32} = a(8000)^{-1}$ or *a* = 2,000,000*N*. After the cost decrease, $\frac{Np}{8} = ap^{-1} = 2000000Np^{-1}$ or $p = \sqrt{16000000} = 4000$ per thousand doorknobs. This problem has the remarkable property that the number of doorknob producers stays the same and the short run effect of a cost decrease is the same as the long run effect.

3. (20 points) Consider the production possibilities frontier for wheat *W* and corn *C*. Nation one has a frontier of $W^2 + 3C^2 = 4$ and nation two has a frontier of $2W^2 + 6C^2 = 4$. Both nations consume equal amounts of corn and wheat.

   a. Under autarky, how much does each nation consume?
   b. What is the marginal cost of corn for each nation?
   c. If the relative price of corn to wheat is 1, will trade occur between the two nations?

a. Under autarky, nation 1 consumes 1 unit of corn and 1 unit of wheat, and nation 2 consumes √1/2 units of corn and √1/2 units of wheat.

b. For nation 1, $W = (4 - 3C^2)^{1/2}$, so $\frac{dW}{dC} = -3C(4 - 3C^2)^{-1/2} = -3\frac{C}{W}$. For nation 2, $W = (2 - 3C^2)^{1/2}$ so $\frac{dW}{dC} = -3C(2 - 3C^2)^{-1/2} = -3\frac{C}{W}$.

c. No. Each nation has the same marginal cost, so there is no benefit to specialization and trading.

4. (10 points) Explain why the supply of labor can be described as "backward bending."

In general, as a worker's hourly wage starts to rise, the worker would substitute more hours of work for leisure because the cost of leisure has risen. But at some point, the income effect may be stronger than the substitution effect, causing the person to work fewer hours and enjoy more leisure time. If one were to graph labor supply with wage on the y-axis, the line would curve backwards.

5. (20 points) Safety regulations require a factory to have 3 workers (y) for every machine (x) it runs. The factory's output is given by $T = 15x^{0.5}y^{0.2}$. If the price of x is p, and the price of y is r, what ratio of p to r would make the required input mix optimal?

$T = 15x^{0.5}y^{0.2}$, so $y = T^5 / 15x^{5/2}$. To find the optimal input mix, minimize cost, C = px + ry. $C = px + r(T^5 / 15x^{5/2})$. The derivative with respect to x is $p – (rT^5 / 6)\, x^{-7/2}$, so cost is minimized when $x = (r / 6p)^{2/7}\, T^{10/7}$. So $x / y = x / (T^5 / 15x^{5/2}) = (15 / T^5)\, [(r / 6p)^{2/7}\, T^{10/7}]^{7/2} = 15r / 6p$. We want x / y to equal 1/3 (since the requirement is that 3x = y), so set 1/3 = 15r / 6p. Then the required ratio of prices is p / r = 15/2. That is, the price of x is 7.5 times the price of y.

6. (10 points) How would you use economic analysis to assess whether movie theater tickets and movie DVDs are complements or substitutes in demand? Name two products outside of the movie industry that may be affected by an increase in the price of movie tickets, and explain how.

To determine if movie tickets and DVDs are complements or substitutes, you would want to know how demand for tickets changes when the price of DVDs increases. (and vice versa) If demand decreases, the two are complements, and if it increases, they are substitutes. Other products affected by an increase in movie ticket prices may include:
King-size packs of Sno-caps, theater popcorn (commonly sold at theaters; would expect demand to drop)
Microwave popcorn (demand may increase if people watch more movies at home)
Restaurant meals (effect depends on whether restaurants are complements or substitutes for watching movies in theaters)