CALTECH

Ec 11 Final Examination
Professor R. Preston McAfee
Spring 2006

Instructions: Open book, open notes, calculators and computers are OK for computations or to read class materials, no collaboration.
Partial credit will be assigned. Please show your work.
You may take this test during any consecutive 4 hour period.
Due June 8 by Noon. Please deposit in Box outside Baxter 111.

1. (15%) You have $800 to invest. You can put it in an account at the Caltech Credit Union, which will give you $1000 after 1 year, and grow 4% per year every year after that. Your other option is to buy a 5-year bond that costs $800 and pays you $100 at the end of each year, plus an extra $10,000 at the end of the 5^{th} year. You discount the future at 15%.
    a. Which is the better investment?
    b. What is the present value (today) if you put the $10,000 received after the 5 years into a Credit Union account with 4% growth? (There is no year one bonus as occurred with an immediate investment.)

2. (15%) Consider two consumers and two goods, X and Y. Consumer 1 has utility $u_1(x_1, y_1) = x_1 + y_1$ and Consumer 2 has utility $u_2(x_2, y_2) = \min\{x_2, y_2\}$. Consumer 1 has an endowment of (1, ½) and Consumer 2's endowment is (0, ½).
    a. Draw the Edgeworth box for this economy.
    b. Find the contract curve, and the individually rational part of it. (You should describe these in writing and highlight them in the Edgeworth box.)
    c. Find the prices that support an equilibrium of the system, and the final allocation of goods under those prices.

3. (10%) A consumer has utility $u(x) = x^{3/4}$ and wealth of $30,000. She is about to take part in a gamble that will give her $5000 if a tossed coin lands on heads, but cost her $5000 if it lands on tails.
    a. What is the certainty equivalent of participating in this gamble?
    b. How much would she be willing to pay to not have to take this gamble?

4.(15%)

		Column	
		L	R
Row	U	1, 3	3, 2
	D	4, 1	2, 4

a. Find all equilibria of the above game.
b. What is the subgame perfect equilibrium if you turn this into a sequential game, with Column going first? With Row going first?
c. In which game does Column get the highest payoff—the simultaneous game, the sequential game when Column goes first, or the sequential game when Column goes second?

5. (15%) Two firms are considering entering a new market. Entering would cost each firm 8. Firm 1 has a slight advantage over firm 2, and it has 0 marginal costs and no fixed cost beyond that to enter. Firm 2's costs are $c_2(q) = q +$ the entry cost. Price is given by $p(Q) = 10 – Q$. Find all payoffs for the simultaneous move game in which each firm can enter or not enter, and find the equilibrium of the game. (Assume a firm gets 0 if it does not enter.)

The following three questions can be answered with a short response. (two or three lines)

6. (10%) Imagine many students are bothered by a band that practices in their house (dorm) at 8 in the morning. Using economic analysis, how would you improve the situation?

7. (10%) If a principal wants to hire agents who are willing to take risks, what type of contract should he offer?

8. (10%) Which type of auction would create the larger winner's curse—English or Dutch?

Answers

1. You have $800 to invest. You can put it in an account at the Caltech Credit Union, which will give you $1000 (total) after 1 year, and grow 4% per year every year after that. Your other option is to buy a 5-year bond that pays you $100 at the end of each year, plus an extra $10,000 at the end of the 5th year. You discount the future at 15%.
    a. Which is the better investment?
    b. What is the present value of putting the $10,000 from the bond (after the 5 years) right into a Credit Union account with 4% growth?

a. Credit Union NPV (in thousands) = $-.8 + 1/1.15 + 1.04/1.15^2 + 1.04^2/1.15^3 + ...$
$= -.8 + 1/1.15\ (1 + 1.04/1.15 + 1.04^2/1.15^2 + ...)$
$= -.8 + 1/1.15\ [1/(1-(1.04/1.15))]$
= approx. 8.291 ($8,291)

Bond NPV $= -800 + 100/1.15 + 100/1.15^2 + ... + 100/1.15^5 + 10000/1.15^5$
$= -800 + 100[(1/.15)(1 – (1/1.15^5)] + 10000/1.15^5$
= approx. $4,507

So the Credit Union account is the better investment.

b. NPV $= -800 + 100/1.15 + 100/1.15^2 + ... + 100/1.15^5 + 10000(1.04)/1.15^6 + 10000(1.04)^2/1.15^7 + ...$
$= -800 + 100[(1/.15)(1 – (1/1.15^5)] + (10000/1.15^5)[1/(1-(1.04/1.15)) – 1]$
= approx. 46,541.02

2. Consider two consumers and two goods, X and Y. Consumer 1 has utility $u_1(x_1, y_1) = x_1 + y_1$ and Consumer 2 has utility $u_2(x_2, y_2) = \min\{x_2, y_2\}$. Consumer 1 has an endowment of (1, 1/2) and Consumer 2's endowment is (0, 1/2).
    a. Draw the Edgeworth box for this economy.
    b. Find the contract curve, and the individually rational part of it. (You should describe these in writing and highlight them in the Edgeworth box.)
    c. Find the prices that support an equilibrium of the system, and the final allocation of goods under those prices.

a. Picture should be of a 1x1 box. Consumer 1's indifference curves have slope -1, and consumer 2's indifference curves are L-shaped with vertices along the 45-degree line.
b. The contract curve is simply the 45-degree line. The individually rational segment is where $1 \geq x_1 \geq 3/4$.
c. It is easiest to find the price ratio graphically, by drawing a straight line from the endowment such that it is tangent to the indifference curves of both consumers at the contract curve. This line will have slope –1, so the price ratio of X to Y is 1. At this price, consumer 1 ends up with (3/4, 3/4) and 2 gets (1/4, 1/4).

3. A consumer has utility $u(x) = x^{3/4}$ and wealth of \$30,000. She is about to take part in a gamble that will give her \$5000 if a tossed coin lands on heads, but cost her \$5000 if it lands on tails.
   a. What is the certainty equivalent of participating in this gamble?
   b. How much would she be willing to pay to not have to take this gamble?

a) The consumer's expected utility is $.5(25{,}000)^{.75} + .5(35{,}000)^{.75} = 2{,}273.53$.
Let x be the certainty equivalent.
Then $x^{.75} = 2273.53$
$x = 2273.53^{4/3} = 29{,}895.16$
b) The expected value of the gamble is .5(25,000) + .5(35,000) = 30,000. The risk premium is 30,000 – 29,895.16 = \$104.84.

4.

		Column	
		L	R
Row	U	1, 3	3, 2
	D	4, 1	2, 4

   a. Find all equilibria of the above game.
   b. What is the subgame perfect equilibrium if you turn this into a sequential game, with Column going first? With Row going first?
   c. In which game does Column get the highest payoff—the simultaneous game, the sequential game when Column goes first, or the sequential game when Column goes second?

a. There is one mixed equilibrium: Row plays U 3/4 of the time and D 1/4 of the time, Column plays L 1/4 of the time and R 3/4 of the time.
b. If Column goes first, the SPE is (R, U). If Row goes first, it is (D, R).
c. When Column moves first, he gets 2. When he moves second, he gets 4. In the simultaneous game, his expected payoff is 5/2. So moving second in the sequential game is best for him.

5. Two firms are considering entering a new market. Entering would cost each firm 8. Firm 1 has a slight advantage over firm 2, and it has 0 marginal costs and no fixed cost beyond that to enter. Firm 2's costs are $c_2(q) = q$ + the entry cost. Price is given by $p(Q) = 10 - Q$. Find all payoffs for the simultaneous move game in which each firm can enter or not enter, and find the equilibrium of the game. (Assume a firm gets 0 if it does not enter.)

If both firms entered, there would be a duopoly. Firm 1's profits would be $(10 - q_1 - q_2)q_1 - 8$. Maximizing this with respect to q_1 gives $10 - 2q_1 - q_2 = 0$. Firm 2's profits would be $(10 - q_1 - q_2)q_1 - q_2 - 8$. Maximizing with respect to q_2 gives $10 - q_1 - 2q_2 - 1 = 0$. Set the two equations equal:
$10 - 2q_1 - q_2 = 10 - q_1 - 2q_2 - 1$
$q_1 = 1 + q_2$

Plugging this back in to Firm 1's FOC gives $q_2 = 8/3$, and then $q_1 = 11/3$.
Firm 1's profits are then: $(30/3 – 19/3)(11/3) – 8 = 49/9$.
Firm 2's profits are: $(30/3 – 19/3)(8/3) – 8/3 – 8 = -8/9$.

If only Firm 1 entered, it would have a monopoly, and profits would be $(10 – q)q – 8$. Maximizing over q, we find $10 – 2q = 0$, so $q = 5$. Then profits are $(10 – 5)5 – 8 = 17$.
If only Firm 2 entered, its profits would be $(10 – q)q – q – 8$. Maximizing gives $10 – 2q – 1 = 0$, so $q = 9/2$. Profits are $(10 – 9/2)(9/2) – 9/2 – 8 = 49/4$.

The simultaneous move game is therefore:

		Firm 2	
		Enter	Not Enter
Firm 1	Enter	49/9, -8/9	17, 0
	Not Enter	0, 49/4	0, 0

The equilibrium is for Firm 1 to Enter and Firm 2 to Not Enter.

6. Imagine many students are bothered by a band that practices in their house (dorm) at 8 in the morning. Using economic analysis, how would you improve the situation?

   The band is creating a negative externality. There are several ways to deal with the problem, including rules about noise, taxes on the band, and subsidies to residents of the house where they practice.

7. If a principal wants to hire agents who are willing to take risks, what type of contract should he offer?

   The principal should offer a contract with a low salary but high commission. This will attract employees who are risk neutral or risk loving and those who have high ability (or overconfidence!).

8. Which type of auction would create the larger winner's curse—English or Dutch?

   A Dutch auction would create the larger winner's curse, because participants do not have the opportunity to learn about other people's values the way they do in the English auction.