CALTECH

Ec 11 Midterm Examination
Professor R. Preston McAfee
Winter 2005

Instructions: Open book, open notes, no collaboration.
Partial credit will be assigned. Please show your work.
You may take this test during any consecutive 4 hour period.
Due February 7, by 5:00 PM. Please deposit in Box outside Baxter 100.

1. (20 points) The demand for slinkies is given by a demand relationship: q = 1200 - p. Many firms exist as potential entrants. Each of the potential entrants has a cost function $C(q) = \frac{1}{2}q^2 + 18$ where q is the output of the individual firm.

a. Compute average and marginal costs as a function of output *q* for an individual firm. What output minimizes average total cost?
b. Graph the long run supply curve for the industry. (You may let the number of firms be a continuous variable for the purpose of answering this question.)
c. Compute: the equilibrium number of firms, the quantity supplied by each firm and the market price of slinkies.

2. (20 points) Producers of pins use metal *X* and labor *L* as inputs, and have an output of $y = \sqrt{XL}$ pins. The quantity demanded for pins is $\left(\frac{p}{a}\right)^{-\varepsilon}$, where *p* is the price of pins and *a* is constant. Labor costs $1 per unit.

a. Assuming the supply of pins is competitive, what is the market demand for metal used for making pins?
b. Suppose that in the short run, labor cannot be increased, but in the long run, new workers can be hired and trained. Metal can be varied instantaneously and sells for $2 per unit. What is the short run supply of pins? Hint: compute the amount of X used by minimizing cost, then compute the equilibrium quantity when supply equals demand.

3. (20 points) Paul can produce 2 car washes per hour, and resolve $\sqrt{H}$ javascript errors in *H* hours. Amy does each task at the rate of one per hour. Amy and Paul each have 5 hours to work. Between them, Amy and Paul need to resolve 4 javascript errors.

A. To maximize the number of carwashes, how many javascript errors should Paul resolve? (Fractions are permitted.)

B. Suppose they have ten hours each, how many javascript errors should Paul resolve? Explain the difference of this answer and part A.

4. (10 points) Large US exports include semiconductors, movies, and grain. Is this consistent with Ricardian theory (comparative advantage)?

5. (20 points) You are considering retiring today, which is January 1. You want to be able to spend $100,000 per year in today's terms. You expect inflation to run 3% per year, so you need $103K next year, $106,090 the following year, and so forth. Your planning calls for a 7% (not inflation adjusted) return on investment. How much do you need to have today to retire and accomplish your goals, under the optimistic forecast that you never die? Please assume that you maintain an investment account which earns interest only if the money is present for a calendar year, and that you withdraw the funds for the coming year on January 1 of each year.

6. (10 points) Coffee and tea are demand substitutes. New coffee plantations in Vietnam increase the supply of coffee. What should happen to the price and quantity of tea traded?

1. Marginal cost is q, average cost is $\frac{c(q)}{q} = ½q + \frac{18}{q}$. This is minimized at q=6, with an average cost of 6. The long run supply is horizontal at an average cost of 6. At a price of 6, market demand is 1194, which requires 199 firms.

2. Let m be the price of metal. Given inputs X and L, production is $\sqrt{XL}$. First, let's find out the demand for X for a given output $q = \sqrt{XL}$. $L = \frac{q^2}{X}$, and costs are $L + mX = \frac{q^2}{X} + mx$. Cost is minimized at $X = \frac{q}{\sqrt{m}}$, and the total cost of production is $L + mX = 2\sqrt{m}q$. Thus marginal cost is $2\sqrt{m}$.

Competitive supply of pins satisfies price = marginal cost, or $aq^{-1/\varepsilon} = 2\sqrt{m}$, or $q = \left(\frac{2\sqrt{m}}{a}\right)^{-\varepsilon}$. This gives the demand for X to be

$$X = \frac{q}{\sqrt{m}} = \frac{1}{\sqrt{m}}\left(\frac{2\sqrt{m}}{a}\right)^{-\varepsilon} = \left(\frac{a}{2}\right)^{\varepsilon} m^{(1-\varepsilon)/2}.$$

The short run supply is just the marginal cost of production, fixing labor. Let q be a quantity, $q = \sqrt{XL}$ so $X = \frac{q^2}{L}$. Then the cost of producing q is $2X + L = \frac{2q^2}{L} + L$, which gives a marginal cost of $\frac{4q}{L}$. This gives the competitive supply for fixed L.

3. Let J be the number of javascript errors assigned to Paul; this takes him J^2 hours and he has $2(5 - J^2)$ car washes. Amy resolves 4-J javascript errors, which takes her $4 - J$ hours, which leaves her with $5- (4 - J) = 1 + J$ hours for car washes, which produces $1 + J$ carwashes. Thus, the total number of car washes is $2(5 - J^2) + 1 + J$. Differentiating this expression and setting the derivative equal to zero yields $J = ¼$.

Part B: The answer is the same. Amy performs 10-(4-J) = 6+J carwashes, while Paul performs $2(10 - J^2)$ carwashes, for a total of $2(10 - J^2) + 6+J$. This has the same solution. This solution equates the marginal cost of javascript errors in terms of carwashes.

4. The US certainly has relatively abundant land, which make exports of grain consistent with the theory. Major inputs into semiconductors are capital equipment and skilled labor. The US has these inputs in abundance, but so does Canada, which doesn't export semiconductors. Finally, inputs into the movie-making business are mobile, so that the Ricardian theory isn't very informative.

5. First, express all amounts in inflation-adjusted terms, so that the interest rate is 4%. To maintain the balance, the value of the balance must grow at 3%. Let *B* be the balance prior to any payout. Then

1.03 *B* = Next year balance = — expenditure + interest + current balance

1.03 *B* = —100K + 0.07(*B* — 100K) + *B,*

or *B* = 1.07 x 100K / 0.4 = 2,675,000

[The question was slightly ambiguous. If you assume that you don't have to pay out in the first year, you get

1.03 X = -103K + .07X + X, or X = 1.03 x 100K / 0.4 = 2575000]

6. The increase in supply of coffee tends to reduce the price of coffee. This reduction in price causes consumers to substitute coffee for tea, reducing demand for tea, causing price and quantity traded of tea to fall.