The Coase Conjecture

Selling all the real estate in Europe

Essentially the Coase conjecture holds that a monopolist compete with future incarnations of himself.

Even though the most profitable course of action is to sell the monopoly quantity immediately, and then never sell again, the monopolist cannot resist selling more once the monopoly profit is earned.

That is, subgame perfection condemns the monopolist to low profits.

The Commitment Solution

The seller's marginal cost is set to zero.

Time is discrete, with periods t=1,2,…

Both the seller and the buyers discount each period at δ.

Market demand is given by q, and is composed of a continuum of individuals.

The commitment solution involves a sequence of prices p_1, p_2,….

A consumer with a value v will prefer time t to time t+1 if

(*) $v - p_t > \delta(v - p_{t+1})$

These equations define a sequence of critical values v_t that make the buyer indifferent between purchasing at t and purchasing at t+1.

$$v_t - p_t = \delta(v_t - p_{t+1})$$

This set of equations can be solved for p_t in terms of the critical values:

$$p_t = (1-\delta)v_t + \delta p_{t+q} = (1-\delta)v_t + \delta((1-\delta)v_{t+1} + \delta p_{t+2}) = \ldots$$

$$(1-\delta)\sum_{j=0}^{\infty}\delta^j v_{t+j}.$$

The monopolist sells $q(v_t)$-$q(v_{t\text{-}1})$ in period t, where v_0 is defined so that $q(v_0)$=0. The monopolist's profits are

$$\pi = \sum_{t=1}^{\infty} \delta^{t-1} p_t q_t = \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta)\left(\sum_{j=0}^{\infty} \delta^j v_{t+j}\right)\left(q(v_t) - q(v_{t-1})\right)$$

$$=(1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty}\delta^j v_{t+j}\right) - \sum_{t=1}^{\infty} q(v_{t-1})\delta^{t-1}\left(\sum_{j=0}^{\infty}\delta^j v_{t+j}\right)\right]$$

$$=(1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty}\delta^j v_{t+j}\right) - \sum_{t=2}^{\infty} q(v_{t-1})\delta^{t-1}\left(\sum_{j=0}^{\infty}\delta^j v_{t+j}\right)\right]$$

$$=(1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty}\delta^j v_{t+j}\right) - \sum_{t=1}^{\infty} q(v_t)\delta^{t}\left(\sum_{j=0}^{\infty}\delta^j v_{t+1+j}\right)\right]$$

$$=(1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty}\delta^j v_{t+j} - \delta\sum_{j=1}^{\infty}\delta^{j-1} v_{t+j}\right)\right]$$

$$=(1-\delta)\sum_{t=1}^{\infty} q(v_t)\delta^{t-1} v_t = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1} q(v_t) v_t.$$

Thus, the optimum level of v_t is constant at the one-shot profit maximizing level, which returns the profits associated with a static monopoly.

The ability to dynamically discriminate does not increase the ability of the monopolist to extract rents from the buyers.

How does the requirement that the monopolist play a subgame perfect strategy affect the monopolist's profits?

Let demand be linear: $q(p)=1-p$.

Consider a game which ends at time T.

Let a_t refer to the highest value customer remaining in the population at the end of time t, so that the set of values remaining at the beginning of time t is uniformly distributed on $[0,a_{t-1}]$, and the quantity purchased at time t is $a_{t-1} - a_t$.

In the last period, the monopolist is a one-shot monopolist, and thus charges the price $p_T=\frac{1}{2}\, a_T$ and earns profits $\pi_T = \frac{1}{4}a_{T-1}^2$.

This can be used as the basis of an induction to demonstrate that

$p_t = \lambda_t\, a_{t-1}$ and $\pi_t = \chi_t a_{t-1}^2$.

The last values satisfy $\lambda_T = \frac{1}{2}$ and $\chi_T = \frac{1}{4}$.

The value a_t is determined by consumer indifference between buying at t and buying one period later, along with the beliefs that the monopolist will follow the equilibrium pricing pattern in the future, so that

$$a_t - p_t = \delta(a_t - p_{t+1}) = \delta(a_t - \lambda_{t+1} a_t),$$

or

$$p_t = a_t\,(1 - \delta + \delta\lambda_{t+1}).$$

Thus,

$$\pi_t = p_t(a_{t-1} - a_t) + \delta\pi_{t+1} = (1 - \delta + \delta\lambda_{t+1})a_t(a_{t-1} - a_t) + \delta\chi_{t+1}a_t^2$$

Maximizing this expression over a_t, we see that the firm chooses p_t to induces a_t satisfying

$$a_t = \frac{1 - \delta + \delta\lambda_{t+1}}{2(1 - \delta + \delta\lambda_{t+1} - \delta\chi_{t+1}}) a_{t-1}.$$

Feeding the formula for a_t into π_t and simplifying gives

$$\pi_t = \frac{(1-\delta+\delta\lambda_{t+1})^2}{4(1-\delta+\delta\lambda_{t+1}-\delta\chi_{t+1}})a_{t-1}^2.$$

We have, at this point, verified the induction hypothesis – p_t is linear in $a_{t\text{-}1}$ and π_t is quadratic, provided p_{t+1} is linear in a_t and π_{t+1} is quadratic. Moreover,

$$\lambda_t\, a_{t\text{-}1} = p_t = a_t\,(1 - \delta + \delta\lambda_{t+1}) = \frac{(1-\delta+\delta\lambda_{t+1})^2}{2(1-\delta+\delta\lambda_{t+1}-\delta\chi_{t+1}}a_{t-1},$$

$$\lambda_t = \frac{(1-\delta+\delta\lambda_{t+1})^2}{2(1-\delta+\delta\lambda_{t+1}-\delta\chi_{t+1}}a_{t-1},$$

and,

$$\chi_t = \frac{\pi_t}{a_{t-1}^2} = \frac{(1-\delta+\delta\lambda_{t+1})^2}{4(1-\delta+\delta\lambda_{t+1}-\delta\chi_{t+1}} = \frac{\lambda_t}{2}.$$

This permits the solution for λ_t in terms of λ_{t+1}.

$$\lambda_t = \frac{(1-\delta+\delta\lambda_{t+1})^2}{2(1-\delta+½\delta\lambda_{t+1})} a_{t-1}.$$

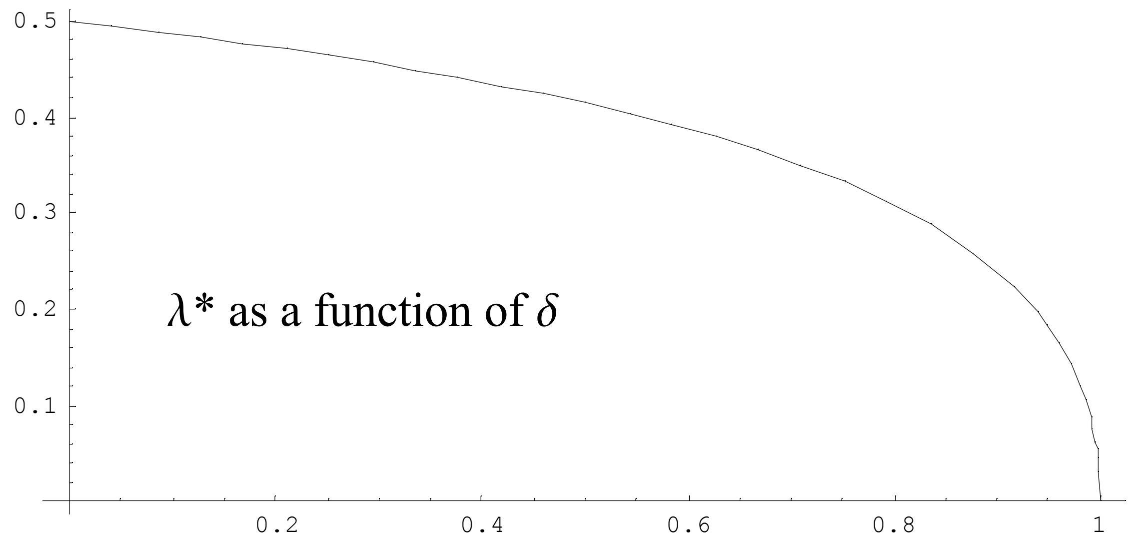
Define $f(\lambda) = \frac{(1-\delta+\delta\lambda)^2}{2(1-\delta+½\delta\lambda)}$, so that $\lambda_t = f(\lambda_{t+1})$.

$f(0) = ½\,(1-\delta)$ and $f(1) = 1/(2-\delta)$.

f is increasing and strictly convex for $\delta \in (0,1)$.

There is a unique fixed point for f, which occurs at

$$\lambda^* = \frac{\sqrt{1-\delta} - (1-\delta)}{\delta} < ½.$$



Since λ_T=½, and f is increasing, the sequence λ_t is increasing in t to ½.

For games with very large values of T, λ_1 is very close to λ^*.

The opening price offered by the monopolist is λ_1, because a_0=1.

The Coase conjecture amounts to the claim that, when the monopolist can cut prices very rapidly, the opening price is close to marginal cost, which was set to zero.

The ability to cut prices very rapidly corresponds to a large discount factor – little discounting goes on between each pricing period.

The Coase conjecture is in fact true, because

$$\lim_{\delta \to 1} \lambda^* = \lim_{\delta \to 1} \frac{\sqrt{1-\delta} - (1-\delta)}{\delta} \to 0.$$

Means of Mitigating the Coase Problem

1. *Other equilibria* don't have this property, but stationary (history independent) ones typically do.

2. *Leasing vs. selling*

3. *return policy or money back guarantee*:

4. *destroy the production facility*

5. *make remaining in the market expensive*

6. *keep the marginal cost secret*

7. *Planned obsolescence*