Handout #1: Price Discrimination

Single unit case. Each consumer demands a single unit, consumers are ranked on a continuum by their type t. Let the distribution of types be F, and index types by their probability $q=F(t)$. Examples of types include age, weight. The willingness to pay of a type t consumer is $p(q)$, which is assumed monotone. Given that p is monotone, p is decreasing without loss of generality. Moreover, marginal costs can be subtracted from p; this makes setting marginal cost to zero without loss of generality. The assumption that p is monotone is not without loss of generality because a discriminating monopolist is going to price based on type, which is assumed to be observable; the monotonicity of p insures that conditioning on observable type is equivalent to conditioning on unobservable value.

A non-discriminating monopolist earns $qp(q)$; let q_0 maximize profits. A two price discriminating monopolist earns $q_1p(q_1) + (q_2-q_1)p(q_2)$ and abuse notation to let q_1 and q_2 stand for the maximizing arguments. Then

Theorem (Varian 1985): Quantity and welfare (sum of profits and consumer surplus) are higher under price discrimination.

> Proof: Note that welfare depends only on quantity due to the single good purchase by consumers. Thus, it is sufficient to prove that quantity is not lower under price discrimination. Suppose not, that is, suppose $q_2 < q_0$. Then $-p(q_0)q_1 > -p(q_2)q_1$. Profit maximization for the non-discriminating monopolist insures $p(q_0)q_0 \geq p(q_2)q_2$. Add these two inequalities to obtain $p(q_0)(q_0 - q_1) > p(q_2)(q_2 - q_1)$, which implies
>
> $$p(q_1)q_1 + p(q_0)(q_0 - q_1) > p(q_1)q_1 + p(q_2)(q_2 - q_1),$$
>
> which contradicts profit maximization of the two price monopolist. Thus, $q_2 < q_0$ leads to a contradiction. Moreover, welfare is strictly higher if either of the optimizations are strict. In particular, if the non discriminating monopolist has a unique optimal quantity, welfare is higher under discrimination.   Q.E.D.

This result suggests that price discrimination is invariably a good thing, but that is not a general result. Suppose there are n markets, and demand is given by $x_i(\mathbf{p})$ in market i where $\mathbf{p}=(p_1,\ldots,p_n)$.

$$\pi = \sum_{i=1}^{n}(p_i - mc)x_i(\mathbf{p}).$$

Marginal cost mc is assumed constant. A non-discriminating monopolist charges a constant price p_0 in all n markets. (Demand might be interdependent because the n markets represent distinct goods sold by the same seller, or because of arbitrage across markets. For example, veterinary and human use of medicines has some limited arbitrage. Methyl-methacrylate is used in both dental and industrial uses and historically experienced costly arbitrage. Identical goods sold in many countries are also subject to limited arbitrage. The discriminating monopolist will charge distinct prices p_i in the markets, $i=1,\ldots,n$.

Define the cross-price elasticity of substitution

$$\varepsilon_{ij} = \frac{p_j}{x_i} \frac{dx_i}{dp_j}.$$

Let E be the matrix of elasticities. Note that, if preferences can be expressed as the maximization of a representative consumer, then the consumer maximizes $u(\mathbf{x})$-$\mathbf{px}$, which gives FOC $u'(\mathbf{x}) = \mathbf{p}$, and thus $u''(x)\mathbf{dx} = \mathbf{dp}$. This shows that demand $\mathbf{x}$ has a symmetric derivative, a fact used in the next development.

The first order condition for profit maximization entails

$$0 = \frac{\partial \pi}{\partial p_i} = x_i + \sum_{j=1}^{n}(p_j - mc)\frac{\partial x_j}{\partial p_i} = x_i + \sum_{j=1}^{n}(p_j - mc)\frac{\partial x_i}{\partial p_j} = x_i\left(1 + \sum_{j=1}^{n}\frac{(p_j - mc)}{p_j}\varepsilon_{ij}\right)$$

Let $L_i = \frac{p_i - mc}{p_i}$, and express the first order condition in a matrix format:

$0 = 1 + E\,L$, and thus $L = -E^{-1}\,1$. This generalizes the well-known one-good case of

$$\frac{p - mc}{p} = -\frac{1}{\varepsilon},$$

where å is the elasticity of demand (with a minus sign). In the one dimensional case, the price/cost margin (aka the Lerner index) is the inverse of the elasticity. (Usually in the one market case, the minus sign is incorporated into the elasticity definition.) In the n market case, the price/cost margins depend on the matrix of elasticities, but still have the simple inverse elasticity form.

In the most frequently encountered version of monopoly pricing, demands are independent, in which case E is a diagonal matrix. The markets are then independent, and

$$\frac{p_i - mc_i}{p_i} = -\frac{1}{\varepsilon_{ii}}.$$

The Robinson-Patman Act of 1933 amended the Clayton act to make price discrimination illegal when the product is sold to intermediaries, rather than final consumers. The prime target of the act was A&P (the Great Atlantic and Pacific Tea Company, which arguably invented the now popular superstore or "category killer"). Does price discrimination increase, or decrease, welfare?

Theorem (Varian, 1985): The change in welfare, ΔW, when a monopolist goes from non-discrimination to discrimination is given by

$$\sum_{i=1}^{n}(p_i - mc)\Delta x_i \leq \Delta W \leq (p_0 - mc)\sum_{i=1}^{n}\Delta x_i.$$

Proof: Let $\mathbf{p_0} = p_0\mathbf{1}$, be the one-price monopoly price vector, and $\mathbf{p}$ represent the prices of the discriminating monopolist. Let v be the indirect utility function (consumer utility as a function of prices). The indirect utility function is convex, and its derivative is demand (Roy's identity). Therefore,

$$\mathbf{x(p_0)(p_0 - p)} \leq v(\mathbf{p}) - v(\mathbf{p_0}) \leq \mathbf{x(p)(p_0 - p)}$$

The change in profits is

$$\Delta\pi = \mathbf{x(p)(p} - mc\mathbf{1}) - \mathbf{x(p_0)1}(p_0 - mc)$$

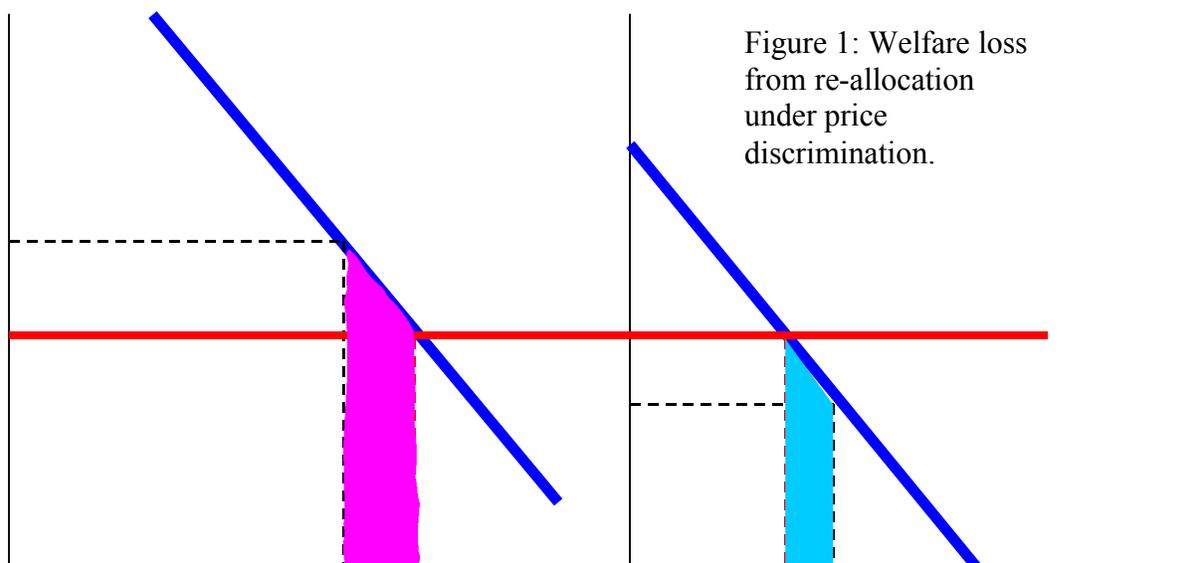Since the change in welfare is the change in consumer utility plus the change in profits, we have

$$\mathbf{x(p_0)(p_0 - p)}+\Delta\pi \leq \Delta W \leq \mathbf{x(p)(p_0 - p)} +\Delta\pi,$$

which combines with $\mathbf{\Delta x=x(p)-x(p_0)}$ to establish the theorem.   Q.E.D.

This theorem has a powerful corollary. If price discrimination causes output to fall, then price

Figure 1: Welfare loss from re-allocation under price discrimination.

Market 1: pink area lost by price discrimination

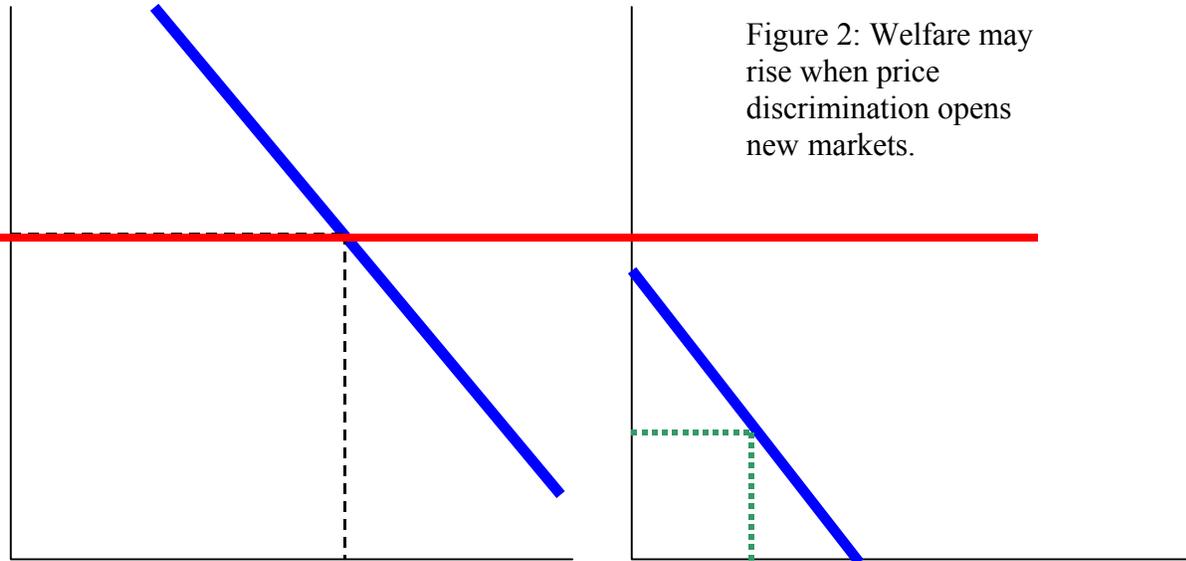Market 2: blue area added by discrimination

Figure 2: Welfare may rise when price discrimination opens new markets.

Market 1: Red line indicates no price discrimination outcome.

Market 2: With price discrimination, market 2 is served.

discrimination decreases welfare relative to the absence of price discrimination. This result, established by Schmalensee (1981) in a more restricted environment, has a simple proof for the case of independent demands. Consider the case of two markets. Price discrimination's effect on welfare is composed of two terms – a change in total output, and a reallocation of the output across markets. The re-allocation always has a negative impact on welfare, because for any given quantity, welfare is maximized by using a single price, because this single price equalizes the marginal value of the good across markets. Thus, price discrimination has a negative reallocation effect; this can only be overcome if the quantity effect is positive, that is, price discrimination induces a higher output.

Even in the simplest two-market case of linear demand, price discrimination may increase or decrease welfare. To see this, first consider the case where the one-price monopolist serves both markets (Figure 1). In this case, it is straightforward to show that the switch to price discrimination leaves the total output unchanged, so that the only effect is the reallocation, which lowers welfare.

It is straightforward to construct cases where welfare rises under price discrimination. Even in the two-market, linear demand case, if price discrimination opens a new market that is otherwise not served, welfare will rise. Indeed, in this case, price discrimination is a pareto improvement, because the monopolist will leave price in the market served under no price discrimination unchanged. That is, price discrimination lowers price in the unserved market, while leaving price in the market served under no price discrimination unchanged. This outcome is illustrated in Figure 2.

Ramsey Pricing

How should a multi-product or multi-market monopolist be regulated? Ramsey investigated this question. Ramsey pricing is the solution to the problem of maximizing social welfare, subject to a break-even constraint for a monopolist.

In particular, consider the problem

$$\max u(\mathbf{x}) - c(\mathbf{x1}) \qquad \text{s.t. } \mathbf{px} - c(\mathbf{x1}) \geq \pi_0.$$

This formulation permits average costs to be decreasing. Write the Lagrangian

$$\Lambda = u(\mathbf{x}) - c(\mathbf{x1}) + \lambda(\mathbf{px} - c(\mathbf{x1})) = u(\mathbf{x}) - \mathbf{px} + (1 + \lambda)(\mathbf{px} - c(\mathbf{x1}))$$

The lagrangian term λ has the interpretation that it is the marginal increase in welfare associated with a decrease in firm profit. Using Roy's identity,

$$0 = \frac{\partial L}{\partial p_i} = \lambda x_i + (1 + \lambda) \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_j}{\partial p_i} = \lambda x_i + (1 + \lambda) \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_i}{\partial p_j}$$

$$= \lambda x_i + (1 + \lambda) x_i \sum_{j=1}^{n} \frac{p_j - mc}{p_j} \varepsilon_{ij}.$$

Write the first order conditions in vector form, to obtain

$$-\lambda/(1+\lambda)\ \mathbf{1} = \boldsymbol{E}\ \mathbf{L}.$$

This equation solves for the general Ramsey price solution:

$$\mathbf{L} = -\frac{\lambda}{\lambda + 1} \boldsymbol{E}^{-1} \mathbf{1}.$$

Note the similarity to monopoly pricing – the welfare optimization problem has the same structural form as the monopoly problem, and moreover the monopoly outcome arises when $\lambda \to \infty$. Setting $\lambda = 0$ maximizes total welfare and sets price equal to marginal cost in all industries. Such a pricing scheme will give the firm negative profits when average costs are decreasing, because marginal cost is less than average costs.

In general, the Ramsey solution is a mixture of marginal cost pricing and monopoly pricing. That is, the Ramsey solution goes part of the way, but not all of the way, toward monopoly pricing. Such a description is at best an approximation, because elasticities are not constant along the path that connects marginal cost pricing with monopoly pricing, but nevertheless, Ramsey pricing generalizes both with a single formula.

Arbitrage

Cross-price elasticities can be interpreted as a consequence of arbitrage by individuals. This arbitrage responds in a continuous way to price changes, and thus obtains a "cost per unit" form of arbitrage. For example, suppose leakage from the low priced market to the high priced market costs $c(m)$, where m is the size of the transfer from market 1 to market 2, and that values in the two markets are otherwise independent. The function c is assumed convex, with $c'(0) = 0$, which insures that goods flow from the low priced market to the high priced market. Finally, assume that consumer demands in markets 1 and 2 are $q_1(p_1)$ and $q_2(p_2)$. The demands facing the seller, x_i will satisfy:

$$p_1 - p_2 = c'(m),$$
$$q_1(p_1) - m = x_1, \text{ and}$$
$$q_2(p_2) + m = x_2,$$

An interesting aspect of these equations is that demand is reconcilable with preferences of a single consumer, that is,

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i}.$$

This equation insures that the analysis of the previous section continues to hold. Thus, in particular, arbitrage does not overturn the welfare results already provided, nor does it influence the inverse elasticity results. Arbitrage does, however, invalidate the independent market model.

Means of Preventing Arbitrage

1. Services

> If someone comes to your home, they can charge you a price based on your home's location; you can't resell the service.

2. Warranties

> A manufacturer may void the warranty if the good is resold; this reduces the value of resale.

3. Differentiating products

> Brazil switched to ethanol for cars. To prevent people from drinking this, they added a little gasoline.

4. Transport costs

> timber, concrete, gravel

5. Contracts

> I get a free textbook provided I don't sell it. I have sold them anyway in the past; these contracts may be expensive to enforce. There are companies that specialize in shaving the "professor's desk copy" off.

6. Matching problem

> No market for someone who needs what you can buy cheap. Airline tickets are an example.

7. Government

> U.S. forces the sale of agricultural products at different prices (e.g. to soviets).

8. Quality

> offer different qualities to segregate the market. high quality sells to high value users, low quality sells to those who can't afford high quality.